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APPLIED MECHANICS Reviews

A CRITICAL REVIEW OF THE WORLD LITERATURE IN APPLIED MECHANICS AND RELATED ENGINEERING SCIENCE

REVS. 1328-1675 VOL. 9, NO. 5 MAY 1956

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Published Monthly by
THE AMERICAN SOCIETY OF
MECHANICAL ENGINEERS
at Easton, Pa., and edited by
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Subscription and Production Office: **The American Society of Mechanical Engineers, 29 West 39th St., New York 18, N. Y., U. S. A.**

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APPLIED MECHANICS REVIEWS, May 1956, Vol. 9, No. 5. Published Monthly by The American Society of Mechanical Engineers at 20th and Northampton Streets, Easton, Pa., U. S. A. The editorial office is located at the Southwest Research Institute, San Antonio 6, Texas, U. S. A. Headquarters of ASME, 29 West 39th St., New York 18, N. Y., U. S. A. Cable address: "Dynamic," New York. Price $2.50 per copy, $25.00 a year; to members of ASME $1.00 per copy, $10 a year. Changes of address must be received at Society headquarters seven weeks before they are to be effective on the mailing list. Please send old as well as new address. . . . By-Laws: The Society shall not be responsible for statements or opinions advanced in papers or printed in its publications (B13, Par. 4). . . . Entered as second class matter, January 11, 1948, at the Post Office at Easton, Pa., under the Act of March 3, 1879.
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APPLIED MECHANICS REVIEWS

VOL. 9, NO. 5 MARTIN GOLAND *Editor* MAY 1956

LINEAR PROGRAMMING

A. J. HOFFMAN

NATIONAL BUREAU OF STANDARDS AND OFFICE OF NAVAL RESEARCH, WASHINGTON, D. C.

ALTHOUGH many of its basic ideas go back to relative mathematical antiquity, linear programming, as a subject with a title and areas of research and application, was defined by George B. Dantzig in 1947. In an attempt to generalize the Leontief interindustry model for purposes of Air Force planning, he observed the desirability of replacing the system of linear equations by a combined system of equations and inequalities. Subsequently, he added a linear objective function to the system; i.e., the goal of the program became to find, among all vectors satisfying the system of equations and inequalities, one which minimized or maximized the objective function. By the fall of 1947, with the aid of ideas of Koopmans, von Neumann, and Hurwicz, the subject was born, along with the "simplex" procedure, to be explained below, for computing answers to specific problems.

Since its creation, the subject has flourished, claiming the attention of a large number of mathematicians and scholars from other fields, for the generality of the model has made it applicable to a variety of situations. The most widely publicized applications have been to industrial and business operations in the context of the amorphously defined "operations research." Such publicity may have created a distorted image of its significance, however. Linear programming is merely a mathematical tool, applicable when its hypotheses are satisfied in a given situation, and it seems to the writer that, in many minds, its important role in operations research has been exaggerated and its usefulness outside of operations research underestimated. We shall try to indicate the scope of these applications, after first summarizing the basic theory and computing procedures.

I. BASIC THEORY

The basic theoretical tool is the fundamental theorem on convex sets: if a point p is not contained in a closed convex set K, then there exists a hyperplane separating p from K. Suitably paraphrased, this lemma may be applied to demonstrate intimate connections between two linear programming problems called "primal" and "dual." The exact form in which this duality is asserted depends on the form in which one canonically poses a programming problem, and there is no general agreement on this. Dantzig generally poses the primal problem as:

minimize

[1] $$c_1x_1 + \ldots + c_nx_n$$

where $(x_1, \ldots, x_n)$ satisfy

[2] $$\begin{aligned} a_{11}x_1 + \ldots + a_{1n}x_n &= b_1 \\ &\cdots \\ a_{m1}x_1 + \ldots + a_{mn}x_n &= b_m \\ x_1 &\geqq 0 \\ &\cdots \\ x_n &\geqq 0 \end{aligned}$$

Then the dual problem becomes

maximize

[1′] $$b_1y_1 + \ldots + b_my_m$$

where $(y_1, \ldots, y_m)$ satisfy

[2′] $$\begin{aligned} a_{11}y_1 + \ldots + a_{m}y_m &\leqq c_1 \\ &\cdots \\ a_{1n}y_1 + \ldots + a_{mn}y_m &\leqq c_n \end{aligned}$$

The duality theorem then asserts that

[3] If there is an $(x_1, \ldots, x_n)$ satisfying [2] and a $(y_1, \ldots, y_m)$ satisfying [2′], then the minimum of [1] is the maximum of [1′].

[4] If there is an $(x_1, \ldots, x_n)$ satisfying [2] and the minimum of [1] exists, then there is a $(y_1, \ldots, y_m)$ satisfying [2′].

[5] If there is a $(y_1, \ldots, y_m)$ satisfying [2′] and the maximum of [1′] exists, then there is an $(x_1, \ldots, x_n)$ satisfying [2].

A somewhat more symmetric form of the duality theorem (indeed, the form in which the duality was first discovered, in part by von Neumann, and in part by Gale, Kuhn, and Tucker) occurs if one considers the primal problem to be

minimize $c_1x_1 + \ldots + c_nx_n$,

where $(x_1, \ldots, x_n)$ satisfies

$$\begin{aligned} a_{11}x_1 + \ldots + a_{1n}x_n &\geqq b_1 \\ &\cdots \\ a_{m1}x_1 + \ldots + a_{mn}x_n &\geqq b_m \\ x_1 &\geqq 0 \\ &\cdots \\ x_n &\geqq 0 \end{aligned}$$

Then the dual has the form

maximize $b_1y_1 + \ldots + b_my_m$

where $(y_1, \ldots, y_m)$ satisfies

$$a_{11}y_1 + \ldots + a_{m1}y_1 \leqq c_1$$

$$a_{1n}y_1 + \ldots + a_{mn}y_m \leqq c_n$$

$$y_1 \geqq 0$$

$$y_m \geqq 0$$

Then [3], [4], and [5], amended in the obvious way, apply to this pair of primal and dual problems. One can easily show that the two duality theorems are equivalent by noting that the set of points satisfying an equation can be represented by a pair of inequalities, and that an inequality

$$a_1x_1 + \ldots + a_nx_n \leqq b$$

can be replaced by

$$a_1x_1 + \ldots + a_nx_n + z = b$$

$$z \geqq 0$$

An important consequence of the duality theorem is that one can solve both primal and dual by solving a large system of inequalities. Consider the primal and dual in the Gale, Kuhn, Tucker form, and let A be the $m \times n$ matrix (a_{ij}). By virtue of [3], any solution of the system of inequalities in the $m + n$ variables $(y_1, \ldots, y_m; x_1, \ldots, x_n)$

$$\begin{pmatrix} 0 & A \\ -A' & 0 \\ b' & -c' \end{pmatrix} \begin{pmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_m \\ x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{pmatrix} \geqq \begin{pmatrix} b_1 \\ \cdot \\ \cdot \\ \cdot \\ b_m \\ -c_1 \\ \cdot \\ \cdot \\ \cdot \\ -c_m \\ 0 \end{pmatrix}$$

solves both problems. Similarly, if the symmetric game defined by

$$\begin{pmatrix} 0 & A & -b \\ -A & 0 & c \\ b' & -c' & 0 \end{pmatrix}$$

has an optimal strategy in which the $(m + n + 1)$th component is positive, then dividing every other component by the last one yields a solution to both programming problems.

In addition to the duality theorem, other basic tools are the fact that a linear function defined on a closed convex set is both concave and convex. It shares with concave functions the property that a minimum, if reached, is attained at an extreme point (= vertex), and it shares with convex functions the property that a local minimum is a global minimum. The second property appears to be more desirable than the first, in that the computing techniques of linear programming can be extended to cover cases in which the function to be minimized is convex, but so far the concave case has not been so extended except in fortuitous circumstances.

2. COMPUTING TECHNIQUES

By far, the most popular algorithm is the simplex method, which has undergone various extensions, notably by Beale, Charnes, Cooper, Lemke, Orden, Wolfe, and Dantzig himself, since its original conception. The algebraic steps are similar to those involved in solving linear equations by Gaussian elimination. Geometrically, it can be conceived as moving along the boundary of a polyhedron from vertex to adjacent vertex until one has gone as far as possible in the optimal direction. A step consists of examining the adjacent vertexes. The number of steps seems to be approximately m if an initial vertex is at hand. If not, one can use any one of various tricks to find an initial vertex, and these seem to be tantamount to an additional m steps.

No one knows an effective theoretical estimate of the number of steps, and it would be reassuring to have such an estimate available to supplement the extensive empirical experience. The success of the simplex method has been based on the observed fact that the number of steps has not been impossibly large.

One can also use methods for solving systems of linear inequalities and methods for solving games in order to solve programs, because of the reasons given in Section 1. Of particular interest are (a) the relaxation method for solving inequalities (suggested by T. S. Motzkin), (b) the fictitious play method for solving games (suggested by G. W. Brown), and (c) a new method of von Neumann for solving games. Relaxation is an iterative process. If a vector $\mathbf{x}^{(i)}$ is at hand, $\mathbf{x}^{(i+1)}$ is the orthogonal projection (or an over- or underextension of it) of $\mathbf{x}^{(i)}$ on a hyperplane π whose associated half-space does not contain $\mathbf{x}^{(i)}$. (In general, π is either the furthest hyperplane or the next hyperplane if the inequalities are arranged in an infinite periodic sequence.) This process converges to some point satisfying the inequalities. Fictitious play consists of alternate "plays" of a game by the two players, each player choosing a pure strategy presuming that his opponent's mixed strategy will be what his accumulated history of choices indicates. Each player's history converges to his set of optimal strategies. Experiments with these two iterative methods indicate they are not as fast as the simplex method, and they have not been used for large practical problems. Indeed, large games and large systems of inequalities are more rapidly solved by the simplex method.

Von Neumann's new method for solving games cannot be so easily summarized. It is an infinite iterative procedure, and the proof of its validity seems to imply that convergence will be as slow as in fictitious play, but no numerical experience is now available. A remarkable feature of the method is that one can easily state from the entries in the game matrix upper bounds for the number of steps needed to achieve a preassigned accuracy.

There are other methods known to converge for solutions of games and inequalities, but numerical experience with them seems unavailable. Indeed, even for the iterative methods tested rather extensively, there has not been enough experimentation with the use of accelerating devices, or with estimating their usefulness when answers of very low precision (or very high precision) are desired, or on use of several methods in combination.

In our discussion of methods, we have so far considered only methods applicable to the general linear programming problem. Many problems arise that are so large that the most powerful general method on the world's fastest computer would require weeks or months to reach an answer, and such problems, if they are to be solved at all, require special methods. The best known of the special problems (it has arisen more frequently in applications than all other linear programming problems) is the following:

$$\text{minimize} \sum_{j=1}^{n} \sum_{i=1}^{m} c_{ij}x_{ij}$$

where

$$x_{ij} \geqq 0 \qquad i = 1, \ldots, m, \; j = 1, \ldots, n$$

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad j = 1, \ldots, n$$

This is called the "transportation problem," because, as originally posed by Hitchcock and Koopmans, the object is to ship a homogeneous commodity from various origins (where supplies a_i were available), to various destinations (with requirement b_j), so that the shipping cost is a minimum. Here c_{ij} represents the unit cost of shipment from i to j. The method most frequently used in practice for solving such problems is an adaptation of the simplex method, but a host of other methods, due to Dantzig, Flood, Fulkerson, Gleyzal, Kuhn, and others await trial. Other special problems have been successfully treated by ad hoc methods, and this search for special techniques for particular situations now claims more research attention than the pursuit of general algorithms.

3. APPLICATIONS

The wide variety of circumstances in which one meets the hypotheses of the linear programming model is striking. The examples selected will be classified as theoretical and practical, where a "theoretical" application is a use of linear programming methods to prove a theorem rather than to reach a numerical answer.

(a) THEORETICAL APPLICATIONS

Despite the fact that the linear programming model presupposes that the variables vary continuously, it has been used successfully in the study of various combinatorial problems, both to unify old results and derive new ones. A typical use of the technique is in the following proof of the well-known theorem that, in a linear graph G in which every cycle is even, the smallest number of arcs whose end points comprise all nodes of G is the largest number of nodes no two of which have an arc in common. Let $a_{ij} = 1$ if the i-th node is on the j-th arc and 0 otherwise. Consider the primal problem: Minimize $\sum_j x_j$, where $\sum_j a_{ij} x_j \geq 1$, $x_j \geq 0$. It is easy to prove that, because every cycle of G is even, every vertex of the convex set is integral; clearly at each vertex, $x_j = 0$ or 1. The minimum occurs at a vertex, and the set of all arcs j such that $x_j = 1$ at the minimizing vertex is the set of arcs we seek. Similarly, the dual problem yields the desired set of nodes. The result sought is then a consequence of [4] of the duality theorem.

In economics, studies of conditions of production and pricing under which an economy is in equilibrium lead naturally (under assumptions of linearity) to the study of systems of linear equations and inequalities in nonnegative variables. Techniques involving duality theorem and generalized fixed point theorems have resolved various thorny questions plaguing economic theory.

In the field of structural design, the study of conditions under which a structure will fail can, in certain circumstances, be handled via linear programming. For example, in the problem of designing a frame to handle static loads, which will consume a minimal amount of material, linear programming not only furnishes a means for computing answers, but also offers as a by-product elementary results about properties of solutions which have interesting physical interpretations.

(b) PRACTICAL APPLICATIONS

These applications include the control of metallurgical processes, electronic equipment design, gasoline blending, production scheduling, agricultural planning, inventory control, warehouse location, aircraft routing, etc. It is difficult to conceive a broad area of science or management in which linear programming would not be relevant. A fair estimate is that, to date, probably a hundred applications of this type have been made. The examples selected are among those the writer knows about and which are capable of easy exposition, not necessarily the most significant.

In the processing of oil arriving from the fields at a refinery, one may combine various grades to produce gasoline of different octane ratings, whose price increases stepwise with the rating. Under linearity assumptions, the problem of arriving at the most profitable product mix becomes a linear programming problem.

The Quartermaster Corps, buying large quantities of a given item for shipment to various quartermaster depots, must award its contracts at least cost to the government. If the amounts that the bidders are willing to make are a_i, the amounts the depots require are b_j, and the unit cost to the government of buying from the i-th bidder for the j-th depot (taking into account transportation cost as well as quoted price) are c_{ij}, then the problem is essentially in the form of the transportation problem discussed above.

Aircraft engines requiring major overhaul are sent from several bases to a depot. In order to fulfill its mission, the base can acquire its engines by utilizing air or surface transport to move its engines to and from the depot, or buy new engines. Of the three possibilities, the speedier method is also more expensive. By assuming additive costs for purchase, air transport, and surface transport, the problem becomes amenable to linear programming.

In planning an adequate diet for poultry that will be least expensive, one knows, for each unit of feed j, the amount a_{ij} of nutrient i it contains. If c_j is the unit cost of j, and b_i is the minimum requirement of nutrient i, the problem is to minimize $\sum_j c_j x_j$, where $x_j \geq 0$ and $\sum_j a_{ij} x_j \geq b_i$.

Knowing the schedule of trips a fleet of tankers must make, it is desired to rent the fewest number of tankers needed to accomplish the schedule. A given tanker may make several trips if, when its unloading at the first destination is completed, there is still time to travel to another port and acquire a load for the second trip, etc. This problem has been solved by an ingenious use of the transportation problem format.

The scheduling of production to meet sales forecasts has been attacked from many viewpoints, depending on the significance of storage and inventory charges, the cost of changing production rates, the presence or absence of capacity limitations, and, especially, the uncertainty of the forecast. The most interesting application of linear programming in this area has been to the production of fuel oil for heating homes. Problems in this field frequently involve the minimization of convex rather than linear functions.

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Linear programming articles appear in various journals, most frequently in

1 *Journal of the Operations Research Society of America*
2 *Management Science*
3 *Naval Research Logistics Quarterly*
4 *Econometrica*

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Letters to the Editor

1328. Re AMR 6, Rev. 691 (February 1953): Strscheletzky, M., Hydrodynamic fundamentals for the analysis of ship propellers.

In the above review I remarked, regarding the results of Strscheletzky's theory, that ". . .the section of the propeller is less cambered than the isolated section in two-dimensional flow in order to produce equal lift (these results do not appear to agree with those from the theory by Ludwieg and Ginzel)." This conclusion follows from Fig. 38. After a recent conversation with the author, it appears that both the figure and the accompanying text are misleading. The figure includes only the curvature which arises from the bound vortexes. The total curvature from both the bound and free vortexes becomes greater than in two-dimensional flow. W. W. Lerbs, Germany

1329. Re AMR 8, Rev. 3822 (December 1955): Von Baranoff, A., Resistance of a tapered body of rotation in accelerated or decelerated movement.

The review of this paper was taken from *Mathematical Reviews* based on an exchange agreement with *MR* without prior notification to the reviewer, Professor Miles. Professor Miles has asked us to publish his review of another paper by Von Baranoff on the same subject in which review Professor Miles explains his altered opinion on the above paper (AMR **9**, Rev. 1503 in this issue). The editor regrets the inconvenience caused the author and reviewer. Ed.

1330. Re AMR 8, Revs. 2502 and 2503 (August 1955): Kämmerer, Flow in a nozzle with frictional resistance; and Szczeniowski, B., The critical flow of gases through nozzles.

There are a few basic misunderstandings, and the undersigned wonders how they might have arisen on such elementary notions.

The first is the second reviewer's opinion (". . .the polytrop is simply a relation between P and v with no implication about variation in specific heat."), which is obviously not true. One cannot pick up from his fantasy whatever formula binding parameters to represent a process, without correlating this formula with basic laws of thermodynamics: in this case, the first law and the equation of state, which always hold true. The polytrop equation is obtained by integration of the first equation of thermodynamics

$$dQ = c_v dT + (1/J)Pdv$$

while eliminating T from equation of state, $Pv = RT$, and assuming $c = (dQ/dT) = \text{const}$ (it must be borne in mind here that it is the matter of constancy or variation of specific heat of an ideal gas, which in an arbitrary process is in general variable and not of c_p and c_v, variable in a real gas but essentially constant in an ideal gas). The following is obtained

$$dT = (1/J)(Pdv + vdP)$$

$$(dQ/dT) = c_v + (RPdv)/J(Pdv + vdP) = c = \text{const}$$

where $(R/J) = c_p - c_v$. Denoting $[(c_p - c)/(c_v - c)] = n$ (c_p, c_v, c, n being constants), this yields $(dP/P) + (ndv)/v = 0$, $Pv^n = \text{const}$, or $[P/(T)^{n/(n-1)}] = \text{const}$, or $Tv^{n-1} = \text{const}$. The above may be reversed. Assume the relation between P and v describing the process is $Pv^n = \text{const}$, where n is a constant. How will this check with the laws of thermodynamics?

$$dQ = c_v dT + (1/J)Pdv = (c_p/R)vdP + (c_v/R)Pdv$$

The specific heat (in general becoming a function of parameters as soon as the process is mathematically defined, e.g., by $P = f(v)$) becomes

$$c = (dQ/dT) = (c_p vdP + c_v Pdv)/(Pdv + vdP)$$

But $P = Cv^{-n}$; $dP = -nCv^{-n-1}dv$; therefore $c = [(c_v - c_p n)/(1 - n)] = \text{const}$. Thus the assumption $Pv^n = \text{const}$ yields the unavoidable implication of the *specific heat being constant.*

The second misunderstanding is Professor Keenan's statement ("...though Q appears to be not necessarily zero in an adiabatic process...") which is, unfortunately, again not true. In an adiabatic process extending from state 1 to state 2, basically $Q_{1,2} = 0$, while it may be (and in general, is) $dQ \neq 0$. The mathematical form of a polytrop in temperature-entropy coordinates is obtainable as follows

$$d\phi = (dQ/T) = c_p(dT/T) - (c_p - c_v)(dP/P)$$

But

$$dP = [(Cn)/(n-1)]T^{1/(n-1)}dT$$

thus

$$d\phi = [c_p - n(c_p - c_v)/(n-1)][(dT/T) = C_1(dT/T)$$

where C_1 is a new constant. Therefore $T = f(\phi)$ is an exponential line, where T is continuously increasing with ϕ increasing if $n > \gamma = (c_p/c_v)$ (or with ϕ decreasing if $n < \gamma$). No turning $(d\phi/dT) = 0$ thus exists, whatever the value of n, which would permit an adiabatic process to be represented by a polytrop, the area beneath the curve, comprised between the two different states 1, 2, being always positive or negative, while for an adiabatic it must be $Q_{1,2} = 0$.

One more misunderstanding is the statement ("The stationary wall is considered to be imparting work to a gas flowing... without suffering any loss of energy."). When a block of concrete is pushed or pulled along the road with a constant velocity, this road, being at standstill, resists with a stationary force of friction; one who pushes the block must apply a force equal and opposite to the former (equilibrium of forces: a law of mechanics); this latter force is being displaced, and therefore one who pushes must impart work. Obviously, the road imparts force but no work. The same applies to any fluid flowing in a channel with "rough" walls; here the work is done at the expense of the internal energy of fluid or of its kinetic energy, or both. The work done degenerates into heat and restores a part (or a whole) of the internal energy, at the higher entropy level (lower pressure), however. In a channel of constant cross section the kinetic energy actually increases slightly at the expense of the internal energy (if the process may be regarded as adiabatic); the temperature should therefore slightly decrease; naturally, the pressure decreases.

It is true that a polytrop may be (and often has been in the past) applied to a nonperfect gas; this will not be, however, either correct theoretically or correspond to an actual physical phenomenon, except if and when the gas is heated or cooled, the dissipative phenomena such as friction, etc., being practically negligible. Even here the exponent n will be subject to variations if the rate of heat exchange during the process is not correctly correlated with the rate of expansion (or compression).

B. Szczeniowski, Canada

Theoretical and Experimental Methods

(See also Revs. 1348, 1353, 1354, 1355, 1363, 1364, 1367, 1418, 1419, 1433, 1485, 1504, 1515, 1601, 1605, 1615, 1651)

☞1331. **Duschek, A., and Hochrainer, A., Basic features of tensor calculus in analytical representation; part III, Applications in physics and engineering [Grundsüge der Tensorrechnung in analytischer Darstellung]**, Wien, Springer-Verlag, 1955, vi + 250 pp. $5.70.

Book is the third volume of a trilogy by the same authors: tensor algebra, tensor analysis, and applications. Authors do not attempt to present a textbook; however, reviewer feels that their book is one of the best published guides for lectures on application of tensor methods. The technique is illustrated by examples from mechanics, elasticity, hydrodynamics, thermodynamics, electricity, magnetism, and relativity theory.

Readers of the AMR will find concise and useful reading material in the first half of the book, which discusses subjects well known by research engineers. Selecting topics which are contained in most graduate level textbooks, authors achieve the purpose of the book, i.e., to show the superiority of the tensorial method over the clumsiness of the scalar and the weakness of the vectorial (e.g., del-operator!) methods. No completeness is attempted in the book; the few, well-selected chapters, however, show unusual thoroughness and exactness. The precise treatment of the principles of rigid body dynamics is exemplary and several critical questions are treated with elegant accuracy on the level of, for instance, Goldstein's celebrated book.

Cartesian tensors are used with few exceptions, which fact makes authors' book less general than the Synge-Schild book, but it offers somewhat more enjoyable reading material. Treatment of elasticity and the theory of viscous fluids corresponds to Truesdell's ideas with considerably reduced generality.

Book is strongly recommended for those who, having the necessary background, wish to enjoy an elegant and clear presentation of tensor methods, or who desire to set up a course in applied tensor analysis for engineering or physics students.

V. G. Szebehely, USA

1332. **Penrose, R., On best approximate solutions of linear matrix equations,** *Proc. Camb. phil. Soc.* **52,** part 1, 17–19, 1956.

In a previous paper, author generalized the concept of inverse of a nonsingular matrix. Here an application is given to statistical problem of finding "best" approximate solutions of inconsistent systems of equations by least squares. Suggestions for computing generalized inverse are given. Y. Luke, USA

1333. **Rutishauser, H., Determination of eigenvalues and eigenvectors of a matrix with the aid of the quotient-difference algorithm** (in German), *ZAMP* **6,** 5, 387–401, 1955.

Two previous papers [title source **5,** 233–251, and 496–508, 1954] describe theory and some applications of the quotient-difference (=*QD*) algorithm. Here is given an extension which allows the determination of the eigenvectors of a matrix. Let $x_1^{(0)}, \ldots, x_n^{(0)}$ be a coordinate system in which A has Jacobs form (such a system may be constructed with methods of C. Lancros or W. Givens). Then the *Q-D* algorithm allows the construction of a sequence of coordinate systems $x_1^{(2\mu)}, \ldots, x_n^{(2\mu)}$, $(\mu = 0, 1, 2, \ldots)$ which converges for $\mu \to \infty$ to the system of the eigenvectors of A.

From author's summary by A. van Heemert, Holland

1334. **Macmillan, R. H., Contractants: a new method for the numerical evaluation of determinants,** *J. roy. aero. Soc.* **59,** 772–773, Nov. 1955.

Author presents novel technique for computation of determinants. An example is presented. Method is useful for evaluation of Routh-Hurwitz stability test functions.

Y. Luke, USA

1335. **Pöschl, Th., A possible improvement of the Ritz method** (in German), *Ing.-Arch.* **23,** 5, 365–372, 1955.

A procedure is outlined for obtaining approximate solutions of boundary-value problems in differential equations. It is essentially the Ritz method with the unknown function satisfying the differential equation at one point or more within the range

of integration. Four examples involving ordinary differential equations are given and solutions are shown to compare favorably with numerical solutions obtained by other methods. No indication is given of the error in the method and so there is no means of assessing the accuracy of a solution.

A. R. Mitchell, Scotland

1336. Gurevich, M. I., On certain solutions of the wave equation (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **97,** 3, 385–386, 1954 (translated from Russian by M. D. Friedman, 572 California St., Newtonville, Mass. 4 pp.).

The wave equation for $\phi(x_1y_1z)$ can be written in the form

$$\frac{\partial^2\phi}{\partial\sigma^2} + \frac{\partial^2\phi}{\partial\delta^2} = \frac{1}{\sinh^2\delta}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right)$$

by a change of variables. The solution $\phi = r\phi_1 + r^2\phi_2 + \ldots$ within the cone $z^2 - x^2 - y^2 = 0\,(z > 0)$ is

$$\phi_n = \sinh^{n+1}\delta\left(\sinh\delta\frac{\partial}{\partial\delta}\right)^n\frac{F_n(\omega) + \overline{F_n(\omega)}}{2\sinh\delta}$$

where $\omega = \delta + i\sigma$. The term $F_n(\omega) + \overline{F_n(\omega)}$ is a harmonic function determined by the boundary conditions of the physical problem. A discussion of the solution of the boundary problem follows.

J. G. Truxal, USA

1337. Gorn, S., The automatic analysis and control of computing errors, *J. Soc. indust. appl. Math.* **2,** 2, 69–81, June 1954.

Any numerical computation is intrinsically subject to errors (e.g., of "round-off" and of "truncation"), the bounds of which must be estimated so that the accuracy of the result can be evaluated. The estimate can be made by a subsidiary computation, and can be used to improve the accuracy of the main computation, either by modification of the results, by permitting the programmer to make changes in the program, or by automatically causing modifications in the program whenever a certain tolerance is exceeded. As an illustration, author considers a single-step method of integrating systems of ordinary differential equations. A modification of Richardson's "extrapolation to zero step size," in which the extrapolation is done locally at each step as the computation proceeds, is presented which produces two approximate solutions bracketing the true solution. As the error bound is produced step-by-step, automatic error control can be introduced by so writing the program that the step length is halved whenever the error bound exceeds a preset tolerance. A numerical example of the integration of a fifth-order system is given.

C. V. L. Smith, USA

1338. Karplus, W. J., The use of electronic analogue computers with resistance network analogues, *Brit. J. appl. Phys.* **6,** 10, 356–357, Oct. 1955.

A method of solving a partial differential equation of the type

$$\Delta^2\phi = f(\phi)$$

by means of direct-current electronic analog-computing units in conjunction with a resistance network is suggested. The method is illustrated by means of a combination setup to solve the ordinary differential equation and initial conditions

$$\phi'' = k\phi^{-1/2},\ \phi(0) = \phi'(0) = 0$$

The computed solution of this illustration is checked against the solution obtained by straightforward integration.

H. J. Ettlinger, USA

1339. Suddarth, S. K., A statistical method for analysis of factor relationships between two sets of data, *Purdue Univ. Agric. Exp. Sta. Bull.* no. 615, 14 pp., Dec. 1954.

A factor θ is determined with given confidence, such that the difference between the elements of one set of data and θ times the elements of another set of data is not significant. Three sampling situations are treated by different methods, two based on the t-distribution and one on the sign test.

P. Kriezis, Greece

1340. Thrasher, L. W., and Binder, R. C., A practical application of uncertainty calculations to measured data, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-205, 4 pp. + 1 fig.

In order to answer questions as to the reliability of measurements, it is desirable to classify two types of measurements: the multiple-sample and the single-sample. The case for multiple-sample measurement is well established, whereas the case for single-sample has not been established. It is proposed that the tester estimate his error—the uncertainty interval—and give odds that the error would be less than the uncertainty interval if the measurement were repeated a large number of times. A numerical example is presented. From authors' summary

1341. Boldyreff, A. W., Determination of the maximal steady state flow of traffic through a railroad network, *J. Operat. Res. Soc. Amer.* **3,** 4, 443–465, Nov. 1955.

The simplest problem considered in this paper concerns a network that consists of an origin, a finite number of intermediate or junction points, a terminal, and a set of arcs of specified traffic capacities (trains per day) connecting these points. The capacity of the network is to be determined, i.e., the maximum steady-state flow of traffic (trains per day) from the origin to the terminal that this network can handle. Author discusses several ways of reducing the number of arcs by one without changing the capacity of the network. Since such reductions may not be possible at all, or not in sufficient number to simplify the problem significantly, the following "flooding technique" is recommended.

The maximum number of trains is assigned to each arc emanating from the origin. The junction points one arc distant from the origin are then treated as new origins, and the maximum number of trains is sent from each point of this set, starting with the point subject to the severest capacity restrictions and scheduling trains in the following order: (1) forward (to junction points two arcs distant from the origin); (2) laterally (to other points of the set); (3) bottlenecked (trains left at a point of the set after all forward and lateral arcs have been saturated). After the entire network has been covered by proceeding in this fashion, bottlenecked trains are returned to the origin. Finally, the validity of the solution is checked by inspection: if a maximum flow has been found, it will not be possible to find a chain of unsaturated arcs extending from the origin to the terminal. Whereas examples of various degrees of complexity are treated in detail, no proof is offered that the technique must furnish the solution.

W. Prager, USA

1342. Ablow, C. M., and Brigham, G., An analog solution of programming problems, *J. Operat. Res. Soc. Amer.* **3,** 4, 388–394, Nov. 1955.

Problem dealt with is very general static programming problem: Minimize function $C(x_1, x_2, \ldots, x_n)$ subject to restrictions $R_k\,(x_1, x_2, \ldots x_n)$, $k = 1, 2, \ldots, m$. By an ingenious dynamical model, authors show that analog computer can be employed to force "dynamic" point to seek solution of problem when such exist. Point $(x_1, x_2, \ldots, x_n)$ is assigned x_i-velocity equal to sum

of velocities in direction of increase of each R_k when that R_k is negative, zero otherwise, plus a small velocity in direction of decrease of C. Authors do not clearly realize or state that method assumes that C and the R_k be monotonic in each variable in some region about $R_k = 0$, $k = 1, \ldots, m$, and that initial point must somehow be selected in such region. Such monotonicity restrictions are always met in linear programming problems. R. F. Rinehart, USA

1343. **von Kármán, T., Foundations of operation research,** *Proc. 3rd AGARD Gen. Assembly*, AGARD AG6/P3, 82–84, Sept. 1953.

1344. **Modenov, P. S., Analytic geometry [Analiticheskaya geometria],** Moscow, Izdat. Moskov. Univ., 1955, 563 pp. 10.80 rbs.

This is a fine textbook for physico-mathematical and mechanico-mathematical faculties of the Russian universities. Content is divided into two main parts: plane analytic geometry with 14 chapters in 121 subtopics, and space geometry with 12 chapters and 109 subsections.

In addition to usual material covered by courses on the subject, volume in question contains excellent and instructive chapters on the theory of geometrical transformations, based upon the fundamental notion of groups. Careful attention is given to analytic geometry in projective plane and space. Book also gives an excellent presentation of the general theory of the second-order curves and surfaces, based upon a consequent use of invariants.

Volume is not overfilled with trivial exercises such as one often finds in courses on analytic geometry. Nevertheless, all substantial and more complicated facts are illustrated by fine worked-out examples throughout the text. There are altogether 251 beautiful (partly unusually original) figures and many instructive tables. Perhaps a brief historical introduction, some literature references, and a good index of matters treated would be desirable in future editions.

Paper is good, print excellent, the price very low.

The work, considered also from the pedagogical viewpoint, is one of the best in the Russian scientific literature. Reviewer recommends it warmly not only to mathematicians but also to engineers and physicists. V. Vodička, Czechoslovakia

1345. **Privalov, I. I., Analytic geometry [Analiticheskaya geometria], 20th ed.,** Moscow, Gosud. Izdat. Tekh.-Teor. Lit., 1955, 299 pp. 6.90 rubles.

This textbook for the Russian technical high schools covers the ordinary basic material needed in technical practice. Content is divided into two parts: plane and space geometry.

First main section consists of 7 chapters with 86 subtopics; the following part has 6 chapters and 59 subsections. Each chapter ends with numerous examples for exercise. There are altogether 431 such examples in the book; the more difficult of them are provided with solutions.

Presentation is clear and vivid, with emphasis on pedagogical approach. Reading is quite easy, nevertheless very useful. Paper and print satisfy all requirements made of a good textbook for students and teachers at high schools.

The book can be recommended as a first course on the subject. V. Vodička, Czechoslovakia

1346. **Horner, J. G., Dictionary of terms used in the theory and practice of mechanical engineering, 7th ed. revised by S. Abbey,** New York, Philosophical Library, Inc., 1955, iv + 417 pp. $6.50.

This technical dictionary in its revised and enlarged edition will be found useful by the practical engineer, draftsman, turner, or even by the do-it-yourself shop mechanic. Its usefulness for scientific and research purposes will, however, be limited by a certain lack of precision. The somewhat loosely formulated definitions do not quite seem to satisfy the requirements placed upon a first-rate dictionary or encyclopedia of the word-definition type. G. Warga, USA

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 1503, 1550, 1662, 1664, 1666, 1667)

1347. **Green, W. G., Theory of machines,** London and Glasgow, Blackie & Son, Ltd., 1955, xxi + 1034 pp. 40s.

Book is intended for comprehensive review for mechanical engineering students preparing for their final examinations in college or for qualification for professional institutions. Numerous examples are given, mostly taken from previous examinations. Many figures and tables are used. Treatment is thorough and rigorous although only undergraduate mathematics is employed. Graphical methods are emphasized. A 15-page table of contents and a 12-page index show the extent of coverage. Book is large since it includes material usually covered in several textbooks. Scope is indicated by the following chapter and section headings: The kinematics and kinetics of a particle: analytical treatment; the kinematics and kinetics of a rigid body: analytical treatment; the kinematics of machines: geometric representation of motion; mechanisms with lower pairs: graphical analysis; the direct-acting engine mechanism; the balancing of machines; governors; valves and valve gear; friction, lubrication, and efficiency of machines; film lubrication; belt friction; brakes and dynamometers and the motion of vehicles; higher pairing: toothed gearing; cams and cam motions; the vibrations of mechanical systems; motion in three dimensions: gyroscopic motion.

M. Goldberg, USA

1348. **Pajares Diáz, E., On the slider-crank mechanism** (in Spanish), *Rev. Cienc. apli.* **9**, 46, Fasc. 5, 413–421, Sept.-Oct. 1955.

"This article begins by examining the path which (in a slider-crank mechanism) a point of the pitman (?) describes where its end moves along a straight line which does not pass through the center of rotation of the crank. For this, a system of rectangular Cartesian coordinates is employed, and the equations which constitute the solution of the problem are obtained. Further, some particular cases are examined, and last, those in which the trajectory of the point whose motion is studied is a straight line located on the axis of the abscissas, serving as a basis for construction of mechanisms frequently used in indicating or recording machines. Finally, a type of ellipsograph is described, based on the findings of this article and which can be used as a tracing or cutting tool as well as for various other purposes." (Author's abstract)

An algebraic study of a well-known elementary mechanism, the author's contribution being to develop the analysis to study the forces and motions critically in the regions of motion nodes. Although making a generalized approach, most of this paper deals with special cases in which the mechanism proportions produce elliptical or straight-line motions. The ellipsograph described does not appear to be novel to this reviewer.

C. E. Balleisen, USA

1349. **Horn, H., Investigation of causes and remedies for valve mechanism wear** (in German), *Maschinenbau-Technik* **3**, 10, 499–506, Oct. 1954.

Paper derives algebraic relations for sliding velocity and for

geometric and kinematic parameters of rocker-arm-operated valves. Moments on valve stem due to eccentric loading and friction between stem and rocker arm pawl are calculated for numerical (diesel) examples. Various remedies for wear at the guide are discussed, including selection of optimum lift and rocker operating angle, roller drive of the stem, straight line linkage, hydraulic actuation, and double guiding of the stem assembly; of these, the last is recommended. G. A. Nothmann, USA

1350. Ashby, S. H., Overdrives: Problems in their application and design, *Instn. mech. Engrs.* Preprint, 11 pp., 1955.

Overdrives are by no means new, having been extensively used in the United States, and, to a lesser extent, in continental Europe, for many years. Their widespread use in Great Britain is comparatively recent, although the American type was fitted to a small number of British cars before World War II.

In America, the Borg-Warner unit appears to hold the field, while on the Continent probably the most popular type is the extra pair of "overspeed" gears fitted into the synchromesh gearbox. British use of the device has consisted mainly of the Laycock-de Normanville unit, although there are other types, such as the Daimler method of changing one of the trains in the Wilson gearbox from a reduction gear to a step-up gear; for example, turning a four-speed into a three-speed and overdrive box. The Handa overdrive is also a British invention using dog engagement for selection, necessitating the use of the clutch pedal, and operated by manifold depression. Another mechanism producing a similar effect is the Eaton two-speed axle for commercial vehicles. This latter is not an overdrive in the strict sense; it should technically be described as underdrive, being a means of varying the gear reduction between the propeller shaft and the rear-axle half shafts. Overdrive is a means of driving the propeller shaft faster than the engine, and while this is perhaps not the most suitable term, it is now universally accepted.

From author's summary

1351. Miele, A., Optimum flight paths of turbojet aircraft (translation of "Traiettorie ottime di volo degli aeroplani azionati da turboreattori," *Aerotecnica* **32,** 4, 1952), *NACA TM* 1389, 47 pp., Sept. 1955.

The climb of turbojet aircraft is analyzed and discussed, including the accelerations. Three particular flight performances are examined: minimum time of climb, climb with minimum fuel consumption, and steepest climb. From author's summary

1352. Radzimovsky, E. I., Eliminating pulsations in chain drives, *Prod. Engng.* **26,** 7, 153–157, 168–169, July 1955.

Servomechanisms, Governors, Gyroscopics

(See also Revs. 1337, 1360, 1552)

1353. Abramson, H. N., Response curves for a system with softening restoring force, *J. appl. Mech.* **22,** 3, 434–435, Sept. 1955.

Periodic solutions of Duffing's equation with a soft spring term exhibit, for small forcing amplitude, branched response curves such that one branch associates *large* amplitudes with *small* forcing frequencies. "The physical significance of these results is as yet inadequately understood." G. Kuerti, USA

1354. Göller, E., Controls with given roots of the basic equation (in German), *Regelungstechnik* **3,** 11, 268–272, 1955.

Author studies a control loop which is described by a homogeneous differential equation of fourth order with constant coefficients. The solutions of this equation for the control variable φ are investigated under the assumption that the roots of the equation are: (1) $z_1 = z_2 = z_3 = z_4 = -p$; or (2) $z_1 = z_2 = -p$; $z_{3,4} = -p + jp$; or (3) $z_{1,2} = z_{3,4} = -p \pm jp$ (p positive and real). Initial conditions are assumed to be $\varphi_0' \neq 0$, φ_0'' proportional to φ_0', and $\varphi_0 = \varphi_0''' = 0$. In all three cases, φ returns to zero in a "desirable" way after an initial disturbance $\varphi_0' \neq 0$.

In the second part of the paper, two particular control problems (control of speed and control of pressure) which lead to fourth-order differential equations are discussed. It is shown that the free design parameters can be chosen such that one or the other of the discussed root systems appear.

I. Flügge-Lotz, USA

1355. Eckhardt, H. D., Graphical aids for frequency response analysis, *J. aero. Sci.* **23,** 1, 83–86, Jan. 1956.

Use of template embodying gain-phase characteristics of some common transfer functions is suggested as means of eliminating some of the work and opportunities for error found in usual frequency-response analysis. Reviewer believes that appeal of such a device is entirely dependent on individual preferences.

L. Becker, USA

1356. Spearman, F. R. J., The derivation and use of aerodynamic transfer functions of airframes, *J. roy. aero. Soc.* **59,** 11, 743–761, Nov. 1955.

Paper is largely expository, dealing with the application of servomechanism analyses to airplane motions. Aerodynamic transfer functions are defined and derived approximately, effects of nonlinearities and cross-coupling being ignored. Applications to open-loop frequency and transient analyses are presented in some detail, as is the closed-loop performance of autopilots. General comments are made concerning the use of analog computers for solving problems in which cross-coupling and nonlinearities are not ignored.

Reviewer believes paper to be of decided importance to beginners, but probably somewhat less so to experts.

H. N. Abramson, USA

1357. Muzzey, C. L., Improving airplane handling characteristics with automatic controls, *Trans. ASME* **78,** 1, 143–152, Jan. 1956.

Paper describes an important mode of airplane flight in which the aircraft is under the joint control of a human pilot and an automatic control system. The first part of the paper describes the interaction of air loads and inertia reactions which govern the dynamic stability of the airplane, and seeks to give a physical picture of the response of the airplane to control and random disturbances. The paper then describes a research project in which the pilot's controls were disconnected from the control surfaces, and separate servos provided both to drive the aerodynamic surfaces and to resist the actions applied to the cockpit controls by the pilot. The surface servos receive signals from cockpit-control motions and from a stability computer which senses airplane-body motions. Airplane response as seen by the pilot can be varied by adjusting the computer, and the pilot is unaware of the control-surface motions caused by the computer. Thus both airplane stability and control feel can be varied independently and easily in flight. Research airplanes so equipped are now being used to map out regions of acceptability for future design in the handling characteristics problem and are, incidentally, demonstrating means for stabilizing existing airplanes which may be deficient in one or more respects.

From author's summary

1358. **Seaberg, E. C., Laboratory investigation of an autopilot utilizing a mechanical linkage with a dead spot to obtain an effective rate signal,** *NACA TN* 3602, 27 pp., Dec. 1955.

The roll automatic pilot investigated operates on a nonlinear principle, termed the frontlash principle, whereby a dead spot is incorporated in the servomotor feedback linkage to obtain an effective rate signal by reducing the phase lag of the servomotor. By application of the frontlash principle, the servomotor feedback linkage improves the servomotor phase response in a manner similar to that which would be obtained with the use of a rate gyroscope. However, the servomotor travel resulting from a given position-gyroscope displacement is decreased when the frontlash feedback linkage is used. Although the present application was for a roll automatic pilot, its application to other control systems appears feasible.

The results of this investigation indicate that the frontlash automatic pilot has promise as a pilotless-aircraft stabilization system. Laboratory tests of the system conducted on a roll simulator show that, in a certain range of simulated aerodynamic parameters, the nonlinear frontlash automatic pilot has a higher degree of stability than a comparable linear system. However, the transition from a stable to an unstable autopilot-aircraft combination appears to be more rapid with the nonlinear system. The results and applications in connection with the roll-simulator tests indicate that there are limitations in applying linear methods of theoretical analysis to systems having nonlinear components.

From author's summary

1359. **Lee, Y. C., Pickles, A. M., and Miesse, C. C., Experimental aspects of rocket system stability,** *Jet Propulsion* **26,** 1, 31–39, Jan. 1956.

By considering compressibility and friction in the feed lines of a liquid bipropellant system, various stabilizing devices are considered for elimination of low-frequency instability. The relative effects of several of these devices (orifice, inductive, venturi, flexible hose, distributed capacitance core) and of different propellants are determined experimentally by pulsing the propellant flow rate and observing the change in decrement rates of the resultant chamber pressure oscillations. Although the inductive venturi and distributed capacitance core were found to have an appreciable stabilizing effect, the greatest stabilizing effect was achieved by using a more reactive fuel. Both Fourier and Mirragraph analyses of the chamber pressure oscillations indicate the presence of several predominant frequencies.

From authors' summary

1360. **Smith, B. N., Perturbation analysis of low-frequency rocket engine system dynamics on an analog computer,** *Jet Propulsion* **26,** 1, 40–45, Jan. 1956.

Paper is concerned with the formulation of a mathematical model of a rocket engine system and the simulation of this model on an analog computer. Perturbation equations and computer circuit are developed for a pump-fed bipropellant system with turbine throttle valve area as the input variable. The frequency response of a typical system and the use of the computer model for estimating the effects of variations in the dimensions of interacting components on the frequency response are discussed.

From author's summary by L. S. Dzung, Switzerland

1361. **Khokhlov, V. A., Velocity characteristics of a hydraulic motor with spool-valve control** (in Russian), *Avtomatika i Telemekhanika* **16,** 5, 421–430, 1955 (translated from Russian by M. D. Friedman, 572 California St., Newtonville, Mass. 14 pp.)

An equation is given which determines the velocity of piston motion of a hydraulic motor with spool-valve control under the action of an external load. The concept of a hydraulic motor with unit dimensions is introduced. A standard set of curves is constructed for such a motor which enables the velocity of the piston motion for each specific motor to be determined as a function of the spool-valve displacement and of the external load by a simple scale conversion.

From author's summary by S. Fujii, Japan

1362. **Levenstein, H., Tie simplicity to power with pneumatic servomechanisms,** *Control Engng.* **2,** 6, 65–70, June 1955.

1363. **Manildi, J. F., Response time in temperature-indicating instruments,** *Jet Propulsion* **24,** 6, 382–384, Nov./Dec. 1954

Vibrations, Balancing

(See also Revs. 1352, 1435, 1494, 1530, 1557, 1562, 1566, 1575, 1665)

1364. **Daymond, S. D., The principal frequencies of vibrating systems with elliptic boundaries,** *Quart. J. Mech. appl. Math.* **8,** 3, 361–372, Sept. 1955.

Equation considered is $\nabla^2\phi(x,y) + \nu\phi(x,y) = 0$. Two problems arising from different boundary conditions (zero deflection and zero normal derivative) are studied. Solution requires finding zeros of modified Mathieu functions. Method of computation, analysis, and tabular values of eigenvalues are presented.

Y. Luke, USA

1365. **Nesitka, A., Vibration of systems with several degrees of freedom** (in German), *Bauingenieur* **30,** 7, 244–248, July 1955.

Free and forced vibrations of lumped parameter systems are described analytically. Graphic solution of fourth-degree secular equation is shown. Author claims that the use of inverse influence numbers is justified by a certain clarity and that they are usually available from previous static analysis. A numerical example shows the effect on resonant frequencies of a particular system when different modes of vibration are taken into account.

J. Hult, Sweden

1366. **Klotter, K., The attenuation of damped free vibrations and the derivation of the damping law from recorded data,** *Proc. second U. S. nat. Congr. appl. Mech.*, June 1954; Amer. Soc. mech. Engrs., 1955, 85–93.

Author derives approximate solutions to the differential equations of the free vibration of systems (having one degree of freedom) subject to certain forms of nonlinear damping. The Kryloff-Bogoliuboff method is used, and the approximations are thus only true if the damping forces are weak.

The restoring force is linear and three forms of damping term, proportional to powers of velocity, displacement, and amplitude, respectively, are treated. For a single term, each form of damping leads to the same first-order linear differential equation for the attenuation of the vibrations; solutions are given. The general case of two damping terms having different exponents, and a special case with three terms are treated in a similar way.

A number of ways of determining the damping law from vibration records for systems with single and double damping terms are given. Methods are available which do not involve trial-and-error procedure.

The approximate solutions have been checked by comparison with exact and numerical solutions available. The methods of determining the damping law have been tested by applying them to vibration records generated by an analog computer.

The mathematical treatment is concise and each step is clearly explained. The assumptions made are clearly stated.

T. U. Matthew, England

1367. Boyce, W. E., Effect of hub radius on the vibrations of a uniform bar, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-44, 4 pp.

Methods are discussed for obtaining upper and lower bounds on the frequencies of a uniform beam, rotating at a constant speed about an axis at one end, and vibrating transversely to the plane of rotation. Previous results are extended to include the case of a nonzero hub radius. Upper and lower bounds on the first two frequencies are given for several ratios of hub radius to beam length, the lower bounds being obtained by an extension of Southwell's method. These show that the frequencies depend almost linearly on the hub radius for various rotational speeds.

From author's summary by L. E. Goodman, USA

1368. Cranch, E. T., and Adler, A. A., Bending vibrations of variable section beams, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-19, 6 pp.

The fourth-order beam equation may be factored into two second-order parts for beams in which bending stiffness and mass per unit length bear a certain relationship. This observation is exploited to obtain exact solutions in terms of Bessel or more elementary functions for variable section beams.

C. M. Ablow, USA

1369. Mettler, E., Nonlinear vibrations and kinematic instability of strings and bars (in German), *Ing.-Arch.* **23,** 5, 354–364, 1955.

Author discusses problems of longitudinally excited lateral vibrations of strings and bars and combined lateral and torsional vibrations of I-beam excited through pulsating bending end moments.

Linearized equation of first problem leads to the Mathieu equation which gives boundaries between regions of stable and unstable motion. In order to get the actual motion, nonlinear effect of longitudinal strain is taken into account, resulting in a vibration of finite amplitude with half the exciting frequency.

Second problem is an example of system with two degrees of freedom leading to equations similar to the Mathieu equation. Linearized equations give regions of unstable motion. When nonlinear term due to longitudinal strain is taken into account, it is shown that when motion is small it consists of one lateral and one torsional mode vibrating with their respective natural frequencies and with frequency sum equal to exciting frequency.

S. Sjöström, Sweden

1370. Cox, H. L., Vibration of certain square plates having similar adjacent edges, *Quart. J. Mech. appl. Math.* **8,** part 4, 454–456, Dec. 1955.

The fundamental frequencies of flexural vibration are determined for thin uniform elastic square plates that have two adjacent edges either clamped or simply supported and that always have the other two adjacent edges free; a pin-point support exists at the intersection of the free edges. Finite difference methods, which simplify the treatment of the free boundaries for definite values of Poisson's ratio, are used in conjunction with extrapolation procedures to obtain the approximate solutions.

From author's summary

1371. Goodman, L. E., and Klumpp, J. H., Analysis of slip damping with reference to turbine-blade vibration, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-80, 9 pp.

Paper gives theoretical treatment of slip damping in a plane joint between adjacent beams held together by uniform pressure. Resulting effect on vibration is confirmed by careful model experiments. Significance for turbine blade vibration problem is largely by implication, because of different geometry. Measurements on slip damping and results of analysis should also be valuable for structures other than turbine blades.

R. P. Kroon, USA

1372. Donaldson, R. M., Hydraulic-turbine runner vibration, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-130, 7 pp. + 4 figs., 1955.

Paper describes an investigation of the vibration characteristics of Francis turbine runners and some experiments to determine the influence of different shapes of runner-bucket discharge edges on the excitation forces. Modifications suggested by the experiments were made to field runners and were successful in eliminating some objectionable vibration.

From author's summary

1373. Niordson, F., Vibrations of turbine blades with loose hinge support, *Acta Polyt., Mech. Engng. Series* **3,** 3, 1–11, 1954.

Paper is concerned with the determination by analysis of the natural frequency of blades in a rotating system where the blades are fastened to the rotor by loosely fitted pins.

Through the action of centrifugal and tangential gas bending forces, the fundamental mode of the loose hinge supported blade is assumed to be a rigid-body pendulum motion with a bending motion superimposed.

The case treated here is the blade with a straight neutral axis with no twist, one principal flexural inertia being considerably smaller than the other. Author presents the analysis of the equations of motion and obtains a solution by successive approximations, since, due to the complexity of the mathematical relationships, a direct solution is impractical. An example is worked out, the results being presented in graphical form.

The particularly important property of this type of support, in making it possible to avoid resonances with multiples of the rotational speed, is clearly indicated.

Reviewer considers this paper to be a valuable contribution to the art, in treating a difficult subject of considerable interest in present design work. E. G. Allen, USA

1374. Jasper, N. H., A design approach to the problem of critical whirling speeds of shaft-disk systems, *David W. Taylor Mod. Basin Rep.* 890, 37 pp., Dec. 1954.

The problem of resonant whirling of propeller-shaft systems is discussed with special emphasis on those factors determining the critical speeds. Several methods for computing the natural whirling frequencies of propeller-shaft systems are presented and discussed. Among these methods are approximate formulas for the fundamental natural whirling frequency which are suitable for direct application by the shaft designer. Computed and experimentally determined natural frequencies are compared. The possible forced whirling motions due to hydrodynamic forces acting on the propeller are discussed.

From author's summary

1375. Balogh, A., A new method to obtain the natural frequencies of torsional vibrations (in German), *ZVDI* **97,** 6, 178–180, Feb. 1955.

Paper applies to torsional vibrations of shaft with finite number of arbitrarily distributed lumped masses. Method essentially consists of progressive elimination of amplitude ratios from equations of motion by way of sequence of frequency functions ultimately leading to frequency determinant. Roots are finally found by plotting against frequency. Procedure does not look neatly systematic. J. H. Greidanus, Holland

1376. **Major, S., The stress analysis and design of foundations for machines and turbines [Gép- és turbinaalapok számitása és tervezése],** Budapest, Bajcsy Zsilinszky ut 22, State Publisher for Technical Books, 1956, 668 pp. + 512 figs. 136 Hungar. Flor.

Difficulties often arise for structural engineers when designing machine foundations. The reason for these difficulties is to be found in the fact that the structural engineers are best trained in statics, while they could rarely find clearly written papers and monographs about the dynamics of structures. Therefore a gap is filled with this book of S. Major, which can easily be understood and is built up very systematically. It is useful not only as a textbook but also as a handbook for structural engineers.

The subject of the chapters are: The theory of vibrations (50 pp.); building materials of machine foundations (30 pp.); testing of soil and description of the convenient machine foundation methods (60 pp.); analysis and measuring of the dynamical stress of soils (30 pp.); structural description and stress analysis of foundations for steam hammers (40 pp.); for crank-shaft driving machines (70 pp.); for turbines (130 pp.); for other machines (20 pp.); methods and equipments for the elimination of vibrations (70 pp.); the dynamics of hydraulic building objects (20 pp.); finally, the author gives detailed numerical examples with a great number of figures for the most frequent cases arising in practice (100 pp.), and thus gives immediate help for designers in the dynamical stress analysis.

The book is not only a compilation but contains the results of the author's personal investigations. Reviewer mentions the excellent typography and the very good figures of the book. It would be desirable to translate the book into English and publish it abroad, too. J. Barta, Hungary

1377. **Rausch, E., Foundations for steam turbines** (in German), *ZVDI* **97,** 29, 1020–1025, Oct. 1955.

1378. **Mudrak, W., Vibration of steam turbine foundations** (in German), *Öst. Ing.-Arch.* **9,** 2/3, 175–181, 1955.

Computation of natural frequency of frame foundations of steam turbines during transverse vibration is given. Author solves foundation structure as three-dimensional unit; therefore, bending deformations of longitudinal beams and torsional deformations of transverse beams are included. Author gives a numerical example and points out that torsional deformations of transverse table beams give natural frequency greater in comparison with the usual method, which supposes table plate as rigid. This phenomenon must be taken into account, because natural frequency of transverse vibration is generally beneath the engine frequency. V. Menel, Czechoslovakia

1379. **Settles, J. C., Application of dynamic vibration absorber to vehicles,** *Proc. second U. S. nat. Congr. appl. Mech.*, June 1954; Amer. Soc. mech. Engrs., 1955, 111–119.

Author analyzes the effect of dynamic vibration absorbers on the linear and rotational motion of a vehicle traveling over a surface having a sinusoidal profile. The equations of motion for the rotational mode are stated and solutions given without proof. The amplitudes of motion of the main and absorber masses are derived for optimum conditions of tuning and damping. Only the main steps in the argument are shown, the algebra being omitted. The methods of deriving the corresponding results for linear motion and of deducing the combined motion are briefly explained.

Two examples based on a 50-ton capacity railway freight car show the limitation of the fixed mass absorber under conditions of varying load and the improvement resulting from maintaining constant absorber/main mass ratio by arranging each absorber as a platform on which a part of the main load can be carried.

The basis for the definition of optimum tuning and damping conditions is not given. It would be helpful if these important steps were more clearly explained. In the examples, the arbitrary decision to tune for mid-load conditions may not do justice to the fixed mass damper, since tuning to a condition nearer to full load might reduce maximum amplitudes.

T. U. Matthew, England

1380. **Döhring, E., Stability of single-track vehicles** (in German), *Forsch. Geb. Ing.-Wes.* **21,** 2, 50–62, 1955.

The linearized differential equations for the motion of a bicycle given by F. Klein and A. Sommerfeld ["Über die Theorie des Kreisels," Bd. IV, Leipzig, 1910] are extended by a more thorough consideration of the front wheel system so that they are also applicable to motorcycles. Since a general discussion of the four roots of the characteristic equation in dependence of the velocity is impracticable, author investigates three typical single-track vehicles, the motor scooter Vespa, the motor cycles Dürkopp MD 150, and BMW R 51/3. The results of the detailed numerical computations confirmed by measurements are as follows: There are three characteristic types of motions superposed: (1) a vibration damped by low and excited by high velocity, (2) a rapid returning of the system when turned out of balance, and (3) a slow returning at low and a very slow deviation at high velocity. We can distinguish three zones of velocity: up to about 20 km/hr we have instability because of vibrations; from 20 to about 40 km/hr the vehicle is theoretically stable; over 40 km/hr there is a slight increase of a caused deviation. Author further regards the effect of a steering damper. All results are obtained under the assumption that the driver is rigidly connected to the vehicle. R. Albrecht, Germany

1381. **Aubuert-Huetz, M., and Huetz, J., Oscillation of mercury in a U tube. Examination of the experimental result** (in French), *C. R. Acad. Sci. Paris* **239,** 23, 1578–1589, Dec. 1954.

Wave Motion in Solids, Impact

(See also Revs. 1453, 1559)

1382. **Miller. G. F., and Pursey, H., On the partition of energy between elastic waves in a semi-infinite solid,** *Proc. roy. Soc. Lond.* (*A*) **233,** 1192, 55–69, Dec. 1955.

Authors consider the problem of the semi-infinite, isotropic, elastic solid in which elastic waves are set up by a circular disk vibrating normally to the free, plane surface of the solid.

Using previously obtained expressions for the displacement field at large distances from the loading area, they compute the power radiated in each of irrotational, equivoluminal, and surface waves. The investigation is then extended to the problem of multiple point sources placed at equal intervals on the circumference of a circle, for which case the power terms are obtained in quadrature. These are evaluated numerically for a three-element radiator.

Paper concludes with a derivation of an integral equation governing the normal stress distribution over the radiator in terms of the (given, symmetric) displacement. In the previous computations this stress had been assumed constant.

H. Deresiewicz, USA

1383. **Conroy, M. F., Plastic deformation of semi-infinite beams subject to transverse impact loading at the free end,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-49, 5 pp.

Paper extends previous work by same author [AMR 6, Rev. 3355], in which plastic-rigid analysis was developed for infinite

beams under transverse, constant-velocity impact loading. Paper also extends work of H. F. Bohnenblust [AMR **3**, Rev. 2203] in which elastic-plastic solution of same problem is presented. As in author's previous work, difficulties arise which prevent obtaining a plastic-rigid solution for a work-hardening material. Results of both rigid ideally plastic and elastic ideally plastic analyses are presented and compared for the particular case of constant moment plus transverse, constant-velocity impact load at free end of semi-infinite beam.

J. W. Dalley, USA

1384. Goodier, J. N., and Jahsman, W. E., Propagation of a sudden rotary disturbance in an elastic plate in plane stress, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-47, 3 pp.

Starting with the plane stress form of the dynamical equations for a circular elastic plate with a hole, authors obtain solutions for the following two cases: (a) sudden application and maintenance of a shear stress τ_0 at the hole; (b) sudden application and maintenance of a tangential velocity v_0 at the hole.

The solution utilizes Laplace transform methods and a procedure developed by Kromm in solving somewhat similar problems. The resulting equations are solved numerically and the final results are given in the form of curves in which velocity and stress histories are shown as functions of time and position.

S. F. Borg, USA

1385. Mar, J. W., Pian, T. H. H., and Calligeros, J. M., A note on methods for the determination of transient stresses, *J. aero. Sci.* **23,** 1, 94–95, Jan. 1956.

Elasticity Theory

(See also Revs. 1375, 1382, 1385, 1400, 1402, 1407, 1414, 1417, 1421, 1425, 1436, 1446, 1674)

1386. Babuška, I., Rektorys, K., and Výcichlo, F., Mathematical theory of plane elasticity [Matematická theorie rovineé pružnosti], Prague, Naklad. Československ. Akad. Věd, 1955, 527 pp. Kes 39.

There is nothing more to be desired, in technical regard, than the excellent Muskhelishvili work on advanced problems of elasticity [AMR **7**, Rev. 2099]. Nevertheless, every mathematician would probably like a more exact formulation of several basic theoretical facts (such as the concept of a finite and infinite plane curve, of an arc, a more detailed discussion of finite and infinite domains, both simply and multiply connected, etc). Explaining Muskhelishvili's ideas in the style usual in rigorous mathematical works is an important task for further development of this powerful technical tool with the cooperation of mathematicians.

The new Czech book under consideration is doubtless a valuable contribution to developing exact mathematical grounds for solving advanced problems of elasticity by means of analytic functions. Two-dimensional theory is not presented in the usual way as the special case of the more general three-dimensional elasticity. On the contrary, it is developed directly in the form of a system of exact definitions and theorems formulated and proved in a quite original way. Each new notion is carefully discussed, and the volume has the character of a decidedly mathematical work.

Material covered is roughly the same as in chaps. II–V of the cited Muskhelishvili book. Titles and brief characteristics of separate parts give a more detailed idea of the content: Chap. I (pp. 15–55) explains, partly by use of abstract sets, the fundamental concepts of a domain, of strain, stress, and displacement. Chap. II (pp. 56–191) presents first of all the strain-stress relations in simply and multiply connected domains. Then comes an excellent statement of the first and second basic problems of plane elasticity in finite regions, and finally, a detailed formulation of the first fundamental problem in an infinite domain. Chap. III deals carefully (pp. 192–365) with the solution of both basic problems reduced to the Lauricella integral equation. Reader's attention is called especially to useful approximate methods for treating difficult cases whose solution cannot be given in closed form. Chap. IV (pp. 366–404) explains the use of conformal mapping methods for solving advanced problems in plane elasticity.

The last chapter constitutes a fine mathematical appendix (pp. 405–514) on necessary facts from the theory of functions and on solving integral equations. Then come references and a carefully made subject index. Paper satisfies all requirements; print is excellent.

Book emphasizes general viewpoints rather than special questions (these will be treated in a second volume). Nevertheless, it also contains many concrete problems, and much of the material probably comes from original research work of the authors. Quite original careful presentation and detailed discussion of each more complicated question make the book easily readable to engineers and physicists, despite its mathematical character.

The Czech literature in applied mathematics and mathematical physics is not rich and this book must be considered as an extraordinary phenomenon in this regard. Its reading is a real enjoyment for everyone interested in mathematics and its applications. Reviewer highly recommends this work to foreign engineers and physicists. V. Vodička, Czechoslovakia

1387. Bezukyhov, N. I., Theory of elasticity and plasticity [Teoriya uprugosti i plastichnosti], Moscow, Gos. Izdat. Tekh.-Teor. Lit., 1953, 420 pp.

This fine course for students at Russian technical universities prepares the reader for a self-reliant study of special works from the field of elasticity and plasticity. It is of a comparatively advanced character with adequate mathematical presuppositions from the theory of matrixes, tensors, and partial differential equations.

In contradistinction to other textbooks on the subject, this work contains a lot of attractive short accounts of the contemporary state in separate branches of elasticity. It also presents several fundamental results concerning some famous special problems solved in our time by the Russian savants, along with the ideas influencing their authors. And more generally, the book tells in detail of the Russian contribution to the development of the subject in question.

Content is divided into 11 chapters with 131 subtopics. The first seven chapters deal with elasticity; then come 3 sections on plasticity; and the last chapter is devoted to a kind of mutual synthesis of elasticity with related modern branches of science (rheology, plasticity, viscoplastic flow, etc.). The text is followed by a rich register of 136 Russian literary sources, a careful nominal index, and a detailed list of matters treated.

Presentation is clear and very attractive; paper and print are excellent. There are altogether 186 exemplary figures throughout the text. A brief account of separate chapters follows. The book opens with two sections of an introductory character: they give the systems of notation applied for fundamental concepts (such as the stress and deformation components) in mathematical theory of elasticity and plasticity, formulation of basic problems occurring in this branch of science, general equations in the mechanics of solids, etc. Chap. III establishes the fundamental equations of elasticity and applies the general theory to two special cases: first to the plane problem and then to the case

of bodies of revolution with a symmetrical stress distribution.

The following main part is devoted to solving some basic problems of elasticity. Considered are the cases of pure bending and torsion, both in rectilinear and curvilinear coordinates (e.g., the well-known problems of Golovin, Galerkin, and Gadolin). Many themes of this section may serve as useful exercises for independent study.

Chaps. V and VI deal with some classical problems of elasticity where it is possible to find exact solutions. Considered are the elastic half-plane and space, Saint Venant problems of torsion, and variational methods in elasticity.

The following main part deals with several important questions tractable by means of various simplifying assumptions and approximate theories only. Attention is paid especially to the theory of plates and to twisting of bars.

Chap. VIII prepares mathematical backgrounds of plasticity in form of basic equations; section IX treats fundamental problems solvable by elementary analytical methods (pure bending and torsion, plastic equilibrium of thick-walled tubes, plastic deformations of a rotating disk, etc.). Chap. X communicates the results reached in solving some special problems of practical importance (in the first place, the research work of Sokolovskiĭ), and the last section brings a synthesis of different methods in elasticity and related branches of the engineering science.

V. Vodička, Czechoslovakia

1388. **Theocaris, P. S., The stress distribution in a strip loaded in tension by means of a central pin,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-34, 6 pp.

Paper contains theoretical solution of the stress distribution in the neighborhood of a hole symmetrically situated in the strip loaded in tension through a rigid pin. Problem is in close relation to practical cases of riveted joints.

Method used for solution is based on the principle of superposition of biharmonic stress functions. Stresses obtained are both tabulated and represented graphically for various values of the ratio of the radius of the hole to the width of the strip. It is shown that the optimum value of this ratio lies near 0.4, when the stress concentration equals the 4.5 multiple of the uniform principal tension in the strip. V. Kopřiva, Czechoslovakia

1389. **Baclig, E. S., and Conway, H. D., Asymmetrical bending of a cylindrically aeolotropic tapered disk,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-20, 4 pp.

The problem of a disk twisted about a diameter by a couple applied to a rigid core is solved for two cases: outer edge clamped and outer edge simply supported. The thickness is allowed to vary, the disk being thick at the core and thin at the edge.

Numerical values are given for the radial and tangential bending moments and for the tangential twisting moments for an oak disk whose outer radius is four times the rigid core radius. The values of the cylindrically aeolotropic oak disk are compared with an isotropic disk. This comparison shows that radial bending moments of the aeolotropic disk are slightly larger while the tangential bending moments are slightly smaller than those in the isotropic disk. However, the twisting moments in the isotropic disk are, in general, twice those in the aeolotropic disk. For the particular oak chosen, the ratio of the radial modulus of elasticity to the tangential one is 2.22, indicating that this material is agreeably anisotropic. N. C. Costakos, USA

1390. **Mortelmans, F., Practical calculation of pieces under double bending** (in Dutch), *Tech. Wet. Tijdschr.* **24,** 8, 197–203, Aug. 1955.

1391. **Conway, H. D., The indentation of an orthotropic half plane,** *ZAMP* **6,** 5, 402–405, 1955.

The contact pressure between an orthotropic half plane and a rigid flat-ended block under a loading is shown to be independent of the elastic constant of the half plane. D. N. Mitra, India

1392. **Jindra, F., One-dimensional problems for a nonlinear stress-strain law** (in German), *ZAMP* **6,** 5, 345–355, 1955.

Author considers a stress-strain law which is obtained from the usual stress-strain relations of linear elasticity by assuming essentially that the shear modulus and the compression modulus are not constants but depend in a speical way on two invariants of the stress tensor. The strain is assumed to be small. Simple extension, inflation of a thick cylindrical tube, and inflation of a thick spherical shell are investigated, and the results are related to experiments with copper.

Reviewer believes that the stress-strain law of the author is too arbitrary and too special. The same experimental results can be predicted by many different theories, and the special deformations considered by the author were investigated earlier for much more general stress-strain laws and for large strain [see C. Truesdell, *J. rational Mech. Analysis* **1,** 125–300, Apr. 1952, and the literature quoted there]. W. Noll, USA

1393. **Boley, B. A., The determination of temperature, stresses, and deflections in two-dimensional thermoelastic problems,** *J. aero. Sci.* **23,** 1, 67–75, Jan. 1956.

Author presents analytic successive-approximation method for the particular integral of linear partial differential equations. Method is applied to examples where special conditions enable approximate complementary function to be found.

D. R. Bland, England

1394. **Buckland, F. F., and Gatzemeyer, J. B., Transient temperature and thermal stress in locomotive gas turbine buckets,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-179, 14 pp. + 1 table.

Authors have made a detailed study of the possible thermal stresses in the buckets of locomotive gas turbines. An electric analog circuit was established and a Goodyear Electronic Differential Analyzer was used to solve the transient electrical problem. Although, as in all fatigue problems, many assumptions had to be made to correlate the calculated fatigue life to operating life, these assumptions were carefully evaluated so that reasonable accuracy limits were obtained. The paper is worth the careful attention of all gas-turbine designers and operators.

E. E. Sechler, USA

Experimental Stress Analysis

1395. **King, J. W. H., and Jenkins, J. E., Mechanical analogy to a plastic hinge,** *Engineer, Lond.* **200,** 5208, 720–722, Nov. 1955.

Paper describes a mechanical device simulating the moment-rotation characteristic of a plastic hinge, based on the resemblance of the axial load-end displacement curve of a pin-jointed strut to the moment-rotation curve of a beam of ductile metal. The analogous plastic hinges are inserted at joints and other appropriate points of a model of a rigid frame. Loading of the model and observation of the mode of collapse facilitate plastic analysis and design of the prototype frame. Reviewer notes that behavior of the analogous hinge in unloading differs radically from that of a plastic hinge, so that the (rare) cases where a plastic hinge disappears in the course of loading would not be simulated by the model. Apart from this minor caution, device seems ingenious and successful. P. S. Symonds, USA

1396. Naghdi, P. M., and Rowley, J. C., An experimental study of biaxial stress-strain relations in plasticity, *J. Mech. Phys. Solids* **3,** 63–80, 1954.

Paper is a welcome addition to the sparse experimental information which provides some basis for judgment of the various mathematical theories of plasticity which have been formulated recently.

The tests described were performed by initially tensioning a thin tube, later subjecting it to torsion while maintaining, increasing, or decreasing the tension. This procedure produced loading paths which should give strain paths all having initial shear moduli equal to the elastic shear modulus, if the material were isotropic.

It was shown by performing such tests on ten specimens of extremely anisotropic thin aluminum tubes that the initial shear modulus was, in fact, much less than that predicted by isotropic plasticity theory.

In a brief discussion, authors state that this behavior cannot be explained completely by invoking an unsymmetrical smooth yield locus, although one having a vertex at the chosen yielding condition might suffice.

Reviewer found this paper most interesting and instructive, especially with regard to the high quality of the experimental work described. One small criticism might be made in that graphs of shear strain versus axial strain were not included. Such graphs would have enabled easier comparison between stress and strain paths, both actual and predicted.

J. M. Alexander, England

1397. Stowe, E. J., A comparison of the reciprocity and interferometer methods of calibrating piezoelectric accelerometers, *David W. Taylor Mod. Basin Rep.* no. 786, 8 pp., May 1954.

Data are presented and discussed which give calibration of a Massa Laboratories accelerometer Model M-127 (modified base) by the reciprocity method and the interferometer method. Possible explanations for the discrepancy between the two sets of data are also presented. From author's summary

1398. Ormsby, R. B., Jr., An angle-of-attack and angle-of-yaw indicator using a direct-reading, simple strain-gage circuit on sting balances, *David W. Taylor Mod. Basin Rep.* no. 885, 10 pp. + 5 figs., Feb. 1954.

Report presents a method whereby the change in angle of attack (or yaw) due to change of applied forces and/or moments on a sting balance may be read directly on any standard strain indicator. The accuracy of this system is limited only by the inherent accuracy of strain-gage circuits and of calibration techniques. The time required for design, gage application, and calibration is about twice that for any one component on a multicomponent balance.

The method does not depend on gravity and will, therefore, work in any plane. From author's summary

1399. Koepcke, W., Determination of bending moments in plates with the aid of an optical mirror system (in German), *Beton u. Stahlbeton.* **50,** 8, 210–216, Aug. 1955.

Optical (reflection) method is presented for determination of bending moments in plates. Model of specially heat-treated and ground plate glass is supported horizontally in a rigid framework and loaded from beneath by calibrated spring spindles or weighted levers. Camera mounted with aperture in plane of raster plate which is mounted above and parallel to model takes photographs of raster pattern as reflected in model surface. Raster pattern consists of rectangular array of mutually tangent small circles of equal diameter. Variations in slope of loaded plate lead to variable shifts of points in reflected image. Reflections of circles thus distort into shapes which are approximately elliptical. Stereocomparison of photos of raster as reflected by loaded and unloaded plate gives data from which can be calculated principal curvatures and their directions. Brief discussion of use of similitude in extrapolating to prototype plate is included. Traveling telescope fitted with scales and small section of raster plate for determining of curvatures without photography is described.

R. W. Gretter, USA

1400. Durelli, A. J., Okubo, S., and Jacobson, R. H., Study of some properties of Stresscoat, *Proc. Soc. exp. Stress Anal.* 12, 2, 55–76, 1955.

Another report on well-known work initiated systematically by senior author to obtain reliable quantitative results from Stresscoat. Thirty-four variables influencing coating properties are isolated. Following 23 are discussed or considered in present paper: grade of Stresscoat used, amount of thinner added, age of liquid coating, humidity of air used in spraying, pressure of air, relative amounts of air and liquid used in gun, distance of gun from surface being sprayed, speed of spraying transverse, number of passes with gun, time elapsed between spray passes, time elapsed between spraying and starting heat-treatment, rate of temperature increase, temperature of heat-treatment, time maximum temperature is maintained, rate of temperature decrease, humidity in heat-treatment oven, temperature during testing, humidity during testing, elapse of time between spraying and testing, repetition of load, speed of load application on specimen, time maximum load is applied, time interval between increments of load.

Strain sensitivity "defined as the minimum strain necessary to crack the coating in a unidimensional state of stress (sensitivity is defined as the inverse of strain sensitivity). The strain sensitivity is determined by using a strip $^1/_4$ in. by 1 in. by 12 in. loaded as a cantilever under a constant deformation and measuring the location of the first crack." Among results found are: Stresscoat of a given number is often not uniform; amount of thinner added to Stresscoat 1205 affected strain sensitivity; coating is probably more sensitive when made from older liquids; big bubbles in coating are avoided by filter in air system to eliminate water; coating thickness is approximately proportional to number of passes with gun; change of sensitivity is of the order of $\pm 10\%$ for Stresscoat 1205 and for a mixture of Stresscoats 1206 and 1207 cured at 75 F for changes in coating thicknesses between 0.002 and 0.008 in.; average strain sensitivity is 10% higher for groups of 12 strips simultaneously cross-sprayed than for strips individually sprayed longitudinally; in a series of tests with varied time elapsed between spraying passes, little change in strain sensitivity occurred for elapsed time greater than 30 sec; increasing time elapsed between spraying and starting heat-treatment increased strain sensitivity; curing temperatures from 75 F to 155 F decreased the strain sensitivity as much as 50% for some coatings; in a single test relatively rapid cooling produced crazing but no significant change in strain sensitivity compared with slow cooling; high humidity during curing apparently raises strain sensitivity; "If all other variables are kept constant, the sensitivity of the coating is inversely proportional to the testing temperature"; sensitivity varied pronouncedly while strips were cooling after removal from curing oven; anomalous behavior in strain sensitivity is possible as a function of elapse of time between spraying and testing; repetition of load appears to decrease strain sensitivity (possibly because of invisible cracks formed by previous applications of load and accented by later applications); time maximum load as applied has little effect on strain sensitivity and no measurable effect after 15 sec; when load was applied, released, and doubled, time intervals between increments of load had little effect on strain sensitivity. W. R. Osgood, USA

Rods, Beams, Cables, Machine Elements

(See also Revs. 1349, 1368, 1369, 1383, 1391, 1409, 1421, 1440)

1401. **Marrone, A. A., Bending in annular sections,** *Proc. Amer. Soc. civ. Engrs.* **81,** Separ. no. 683, 17 pp., May 1955.

Relationships were derived for the elastic stress distribution within a sector of a thin annulus subjected to an eccentric compressive load. They were then extended to include the stress distribution within a reinforced-concrete pipe.

Author also presented a number of graphs which should be useful to the designer. B. W. Shaffer, USA

1402. **Barta, J., On the estimation of the torsional rigidity of thin-walle d multicellular bars** (in French), *Acta Techn. Hung. Budapest* **12,** 3/4, 333–338, 1955.

Following the treatment by P. Blaise of the same subject [AMR **6,** Rev. 1865], author proposes a formula which establishes the lower and upper limits for the torsional rigidity of the bars with thin-walled and multiply connected cross sections. A brief proof for the proposed formula is given. The application is illustrated by a simple numerical example. The simultaneous equations are solved by iteration. D. H. Cheng, USA

1403. **Curry, W. T., Laminated beams from two species of timber, theory of design,** *Forest Prod. Res. Rep.* no. 10, Dept. scient. indust. Res., England, 28 pp., 1955.

Design procedures are advanced for laminated timber beams, which give consideration (1) to the specific properties of the particular wood species used and (2) to the fact that wood is stronger in tension than in compression. By placing dense laminations where maximum stresses are encountered, that is, at the tension and compression surfaces of the beam, and less dense material between the outer laminations, maximum strength can be obtained for a given cross section with minimum material requirements.

Test data are presented to confirm the design theories advanced.

Similar design procedures were previously discussed by A. G. H. Dietz (USA). However, no reference to the respective publication is made by the author. E. G. Stern, USA

1404. **Lowell, C. M., A rational approach to crankshaft design,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-57, 19 pp. + 11 figs.

Author presents results of strain measurements, using SR-4 gages, on several quarter-size model crankshafts of the type having overlapping crank pins and journal, fillet radii in all cases being $^1/_{16}$ times crank-pin diameter. Crankshafts were tested as simply supported beams with varying center distances, and stress-concentration factor was found to vary with stiffness. Based on test results, an empirical formula is derived and charts are given for finding stress-concentration factors as functions of crankshaft dimensions. Because of large gage length of SR-4 gages used, author's stress-concentration results do not refer to peak stress values in fillets; this is confirmed by comparison with other published data. Author suggests that further research along these lines would be desirable. A. M. Wahl, USA

1405. **Hasselgruber, H., and Schwinges, M., Approximate method for the determination of maximum alternating stress in shafts and in couplings when passing through torsional critical speeds** (in German), *Forsch. Geb. Ing.-Wes.* **21,** 4, 118–122, 1955.

Authors derive a simple method for estimating the maximum alternating stress present in shafts and coupling arrangements if the vibrating system passes a critical speed. The method is explained for undamped systems as well as damped, and leads always to larger values than the exact calculation worked out by F. M. Lewis [*Trans. ASME* **54,** 253–261, 1932]. According to numerical comparisons, differences will be at most 15%, so that the easily applicable formulas may be useful.

K. Poschl, Germany

Plates, Disks, Shells, Membranes

(See also Revs. 1370, 1371, 1384, 1389, 1422, 1424, 1434, 1435, 1438, 1448, 1451)

1406. **Kaul, R. K., and Cadambe, V., A simple approximation for plate deflection problems,** *J. roy. aero. Soc.* **59,** 778–780, Nov. 1955.

When applying Rayleigh-Ritz to plate-bending problems, an approximate shape for the surface is assumed which satisfies boundary conditions and makes potential energy of plate in the equilibrium position a minimum. Author shows that labor is reduced for plates with clamped and simply supported edges by using bar eigenfunctions. Since the coefficients along the principal diagonal of the resulting matrix are preponderant, the effect of other terms in the series is negligible.

Examples are given for a square plate with (a) all edges clamped, (b) two adjacent edges clamped and others simply supported, using bar eigenfunctions for vibrating clamped-clamped and clamped simply supported beams, respectively.

H. D. Conway, USA

1407. **Koiter, W. T., On the diffusion of load from a stiffener into a sheet,** *Quart. J. Mech. appl. Math.* **8,** part 2, 164–178, June 1955.

A rigorous solution is obtained for the diffusion of load from a semi-infinite stiffener into an infinite or a semi-infinite sheet. A Mellin transformation, applied to the basic singular integro-differential equation of the problem, yields a difference equation in a strip of the complex plane which is solved by means of Laplace transforms. The final solution is obtained in the form of an inverse Mellin integral, which is evaluated by contour integration.

From author's summary by Ch. Massonnet, Belgium

1408. **Mohammed, I. A., and Popov, E. P., Effective stiffness of a plate subjected to a local edge moment,** *Proc. second U. S. nat. Congr. appl. Mech.,* June 1954; Amer. Soc. mech. Engrs., 1955, 423–426.

Using familiar methods, authors have derived the rotation of a plate simply supported at $x = 0$, $x = a$ due to a bending moment uniformly distributed along a line parallel to the y-axis. For the particular case of a plate of infinite length with the line moment applied at one edge, they have derived coefficients for the effective width of a beam having the same stiffness coefficient, based on (1) the rotation at the center of the line moment and (2) on the rotation at the ends of the line moment.

The object is apparently to estimate restraint exerted on floor beams by bearing walls into which they are built monolithically, but conditions assumed in analysis would seldom be realized in practice.

A note is added on effect of rectangular cutouts in the slab on restraining stiffness, based on experimental work, which will be reported fully elsewhere. A. Burn, Australia

1409. **Lindner, H., The restraint of reinforced concrete slabs by torsionally rigid edge beams** (in German), *Beton u. Stahlbeton.* **50,** 10, 264–267, Oct. 1955.

Author shows how the moments in the slabs can be calculated if the torsional rigidity of the edge members and the concentrated

loads due to vehicles are included, provided certain simplifying assumptions are made. The resulting moment distributions between supports do not differ greatly from the distributions computed by the well-known method of Saeger ["Ueber die Berueecksichtigung der Torsionssteifigkeit der Randbalken von Stahlbetondecken"; title source p. 230, 1950], but in the case of moments over supports and torsional moments the results appear to be considerably different.

From author's summary by F. M. Mueller, USA

1410. Hodge, P. G., Jr., Stress distributions in nonsymmetric rotating rays, *J. appl. Mech.* **22,** 3, 311–316, Sept. 1955.

The stresses in symmetric or nonsymmetric rotating ray are determined analytically. The method is illustrated by means of examples. The speeds for beginning elastic flow and for large-scale deformation are computed. The latter speed, however, is very laborious to determine, even for simple ray shapes.

An indication is given on how to solve the stress distribution in a nonsymmetric rotating disk according to similar methods. Author promises to report some investigations into that problem elsewhere. I. Jung, Sweden

1411. Yeh, G. C. K., Bending of a rectangular plate on an elastic foundation with two adjacent edges fixed and the others free, *Proc. second U. S. nat. Congr. appl. Mech.*, June 1954; Amer. Soc. Mech. Engrs., 1955, 375–380.

The Ritz energy method is used to obtain an approximate solution for the bending of a thin rectangular plate, fixed at two adjacent edges and free at the other two edges, which is supported by an elastic foundation. The solution applies to general distributions of lateral loads and foundation stiffnesses. It also includes the solution for a plate without foundation as a limiting case. Tables of coefficients necessary in setting up the simultaneous equations are given. Two numerical examples for square plates are presented and, in one case, the result checks with that obtained by another method.

From author's summary by A. Burn, Australia

1412. Haythornthwaite, R. M., The deflection of plates in the elastic-plastic range, *Proc. second U. S. nat. Congr. appl. Mech.*, June 1954; Amer. Soc. mech. Engrs., 1955, 521–526.

Deflections of plates with circular symmetry are computed for an elastic-plastic material that obeys yield condition of Tresca and associated flow rule. At any point on plate entire thickness is assumed either fully elastic or fully plastic. An annular plate simply supported at outer radius and clamped to a centrally loaded rigid disk is analyzed in detail. Testing of steel plate is described and results compared with theory.

From author's summary by P. C. Dunne, England

1413. Cooper, R. M., and Shifrin, G. A., An experiment on circular plates in the plastic range, *Proc. second U. S. nat. Congr. appl. Mech.*, June 1954; Amer. Soc. mech. Engrs., 1955, 527–534.

Nine simply supported mild steel circular plates were loaded well into plastic range under concentric uniformly distributed load. Deflection, strain, and slope data were obtained as functions of load. Comparison is made with the predicted limit loads of Hopkins and Prager.

From authors' summary by P. C. Dunne, England

1414. Seide, P., On the torsion of rectangular sandwich plates, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-40, 4 pp.

Saint Venant's torsion theory is used to obtain the torsional rigidity of a rectangular sandwich plate of constant thickness. The expression consists of three parts, i.e., the twisting stiffness due to the orthotropic core alone, the isotropic faces, and the effect of interaction between the faces and core. Author compares the present solution and the simplified twisting stiffness constant of sandwich-plate theory, using a numerical example. He concludes that the latter expression may be very unconservative when width-core depth ratio is low. In the example worked, with a width-core depth ratio of 10, the degree of unconservation was found to be about 61%. T. T. Loo, USA

1415. Zaid, M., Symmetrical bending of circular sandwich plates, *Proc. second U. S. nat. Congr. appl. Mech.*, June 1954, Amer. Soc. mech. Engrs., 1955, 413–422.

General solutions are obtained for a circular sandwich plate under arbitrary symmetrical loading and boundary conditions. Bending of the face layers is considered in a manner similar to that by N. J. Hoff for rectangular plates. The case of a simply supported plate is examined in detail. For a constant pressure load, this solution is compared to E. Reissner's work in which the face layers are considered as membranes. For most practical cases, the bending factor is negligible. Where this is not true, the results of this paper should be used.

From author's summary by H. J. Plass, Jr., USA

1416. Aschenbrunner, R., and Lange, H. A., Contributions to the calculation of isotropic circular cylindrical shell ceilings (in German), *Bauingenieur* **30,** 5, 171–174, May 1955.

For the purpose of solving the marginal problem for the longitudinal edges of shells there have been tabulated the integration constants for types of shells lying between the long-span shell and the short-span shell specified by Aas Jakobson in *Bauingenieur* 20, p. 394, 1939. G. Sonntag, Germany

1417. Weibel, E. S., The strains and the energy in thin elastic shells of arbitrary shape for arbitrary deformation, *ZAMP* **6,** 3, 153–189, 1955.

Instead of using tensor calculus, which has proved very successful in shell problems and allows deriving strain from the displacement components, author uses vectorial symbols and represents strain by variations of fundamental forms of the classical differential geometry. After formulating strain and kinetic energy, he proposes Lagrange's method. As applications, some relations of oblique plates, cylindrical and helical shells are derived and controlled by rigid-body displacements and rotations. finally, he gives an approximate solution for the resonance frequencies of propeller blades. H. Neuber, Germany

1418. Vekua, I. N., On the solution of the boundary problems of shell theory (in Russian), *Soobshch. Akad. Nauk Gruzin. SSR* **15,** 1, 3–6, 1954 (translated from Russian by M. D., Friedman, 572 California St., Newtonville 60, Mass., 5 pp.).

If an isometric system of coordinate lines on the middle surface of a thin shell is chosen, the differential equations for displacement components differ from the corresponding ones for a plate only in that some terms must be added with displacements and their derivatives of lower order. Method outlined in the paper gives solution of differential equations for clamped edges, but it can be extended to other boundary conditions, especially if the displacement component w along the surface normal appears alone in two conditions and the components u and v in the remaining two. Displacement components are taken as definite integrals over the middle surface, containing the influence functions of a plate with the same contour, the load components, and terms of displacements together with their first and second-order deriva-

tives. Thus a system of integral equations is obtained to which the Fredholm theory can be applied. Method yields unique solutions if the corresponding homogeneous boundary-value problem has only trivial solution $u = v = w = 0$, and equations can be solved by successive approximations if the shell is not too much curved. The solving of equations of a general shell is thus reduced basically to the problem of plates.

Method is first of all very suitable for investigating questions of existence, but, nevertheless, it would be of great interest to apply it to numerical computation in such cases where the successive solution of integral equations and the conformal transformation of the shell contour to shapes with known influence functions can be effectuated with a tolerable amount of work.

A. Kuhelj, Yugoslavia

1419. **MacNeal, R. H., Electrical analogies for stiffened shells with flexible rings,** *NACA TN* 3280, 35 pp., Dec. 1954.

Paper presents an all-electric analog for a stiffened shell of a type like that in an airplane fuselage; i.e., it consists of an outer skin supported on longitudinal stringers which are in turn supported on rings. The analogy contains certain simplifying assumptions of a practical nature, the main ones being: (1) the axis of the structure is a straight line; (2) rings lie in planes normal to the axis of the structure and exert only forces which lie in their own planes; (3) a stringer cannot support bending loads, and its change in direction when passing over a ring is small compared to the radius of curvature of the ring; and (4) the skin can exert only shear forces.

Author has succeeded in producing an almost complete analogy within the framework of the assumptions made. He did this by writing elementary equilibrium and constitutive equations for well-chosen finite portions of the structure, replacing these by finite difference relations, and then constructing the analogy with a minimum of further mathematical manipulations. Author is to be congratulated for appreciating and understanding the fundamental role played by perfect couplers (which become perfect transformers in the analogy) in problems of this sort.

The analogy is incomplete in the following sense. The analog contains resistors, transformers, and generators, which is a dissipative network, yet the mechanical system is purely elastic and hence contains no dissipative mechanism. A complete analogy could be produced and would be a necessity if transient effects are to be examined rather than static equilibrium conditions for which the analogy was designed. If interest centers only in equilibrium conditions, the incomplete analogy presented is simple and economical for practical reasons.

The note is recommended reading for those workers who are faced with making calculations on real structures of the sort described.

H. M. Trent, USA

Buckling Problems

1420. **Gerard, G., Generalized crippling analysis of formed sections,** *J. aero. Sci.* **23,** 1, 88–90, Jan. 1956.

Due to the unusual complexity of the theoretical analysis of plates with large deflections, especially in the plastic regime, many authors have, in recent years, attempted to find empirical formulas for the ultimate strength of a thin-walled short column. Author plots maximum stress for Z-section and channel section columns on log paper against an empirical parameter. Formula thus obtained by drawing a straight line through test data is assumed to be applicable to cross sections of arbitrary shape with two corners. A modified formula is given for sections with more than two corners. A coefficient is introduced to allow for the increased strength in the corners.

There are now available so many formulas and methods for determining the ultimate strength of a thin-walled column that the engineering designer is somewhat confused about which to use. Reviewer's opinion is that there is a serious need for a program of theoretical research work on this problem, perhaps along the lines indicated by Stowell's paper [*NACA TN* 2020, Jan. 1950; AMR **3,** Rev. 1888] on the cruciform column.

S. U. Benscoter, USA

1421. **Loo, T.-T., Effects of large deflections and imperfections on the elastic buckling of cylinders under torsion and axial compression,** *Proc. second U.S. nat. Congr. appl. Mech.*, June 1954; Amer. Soc. mech. Engrs., 1955, 345–357.

A large-deflection theory for thin circular cylinders in torsion is presented, which reveals most of the characteristics observed in tests. The powerful effect of the large-deflection terms on the postbuckling resistance is shown.

A new formula is proposed for initial defects in actual specimens, the considered cause for scattered and low buckling strengths. This formula is applicable both to torsion and to axial compression in thin cylinders. From this, new theories having identical assumptions are developed. The results are in fair agreement with experiment. They are also shown to be in good agreement, when modified, with previous small-deflection and large-deflection theories.

From author's summary by H. J. Plass, Jr., USA

1422. **Cox, H. L., and Klein, B., The buckling of isosceles triangular plates,** *J. aero. Sci.* **22,** 5, 321–325, May 1955.

Elastic critical buckling loads are obtained for isosceles triangular plates with both simply supported and clamped edges loaded with (a) thrust on all edges, and (b) thrust on base and shear on sides. The collocation method with trigonometric approximating series is used and results are shown to be in close agreement with previously obtained solutions. Results are conveniently presented as curves, giving the coefficient β as a function of the altitude-base ratio. The critical load per unit length (on the base for those cases having shear on the sides) can be computed by the equation $N_{CR} = \beta D/h^2$, where D is the plate stiffness and h the altitude.

C. T. West, USA

1423. **Besseling, J. F., Analysis of the plastic collapse of a cruciform column with initial twist, loaded in compression,** *J. aero. Sci.* **23,** 1, 49–53, Jan. 1956.

The large influence of very small, and therefore unavoidable, imperfections upon the ultimate compressive load of a cruciform column according to the incremental or flow type of mathematical theory of plasticity has been demonstrated by Onat and Drucker. In this paper the process of plastic collapse of a cruciform column with initial twist is further analyzed. In contrast to the model of Onat and Drucker, the flanges are treated here as solid plates. The ultimate compressive stress is determined by a step-by-step procedure, using a simple isotropic flow theory. It is found that collapse is not connected with an ordinary buckling failure, but is caused by a drop in the compressive stress for compressive strains exceeding the critical strain and accompanied by not yet very fastly growing twist. This drop is due to the interaction of compressive strain and shear strain in the plastic stress-strain relations.

From author's summary by G. Gerard, USA

1424. **Dimaggio, F. L., Effect of an acoustic medium on the dynamic buckling of plates,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-35, 6 pp.

Paper is motivated by the behavior of plates, forming the hulls of ships, being subjected to sudden compressive loading. The problem is idealized by considering an infinite plate on one side of which is an infinite body of acoustic fluid. The plate has sup-

ports which run transverse to the direction of loading. Exact solutions for the general case of a compressible fluid are obtained by means of the Laplace transform, and the plate response determined. Approximate solutions are then deduced for an incompressible fluid, and it is shown that the effect of such fluid can be accounted for by a virtual mass concept. A numerical example is worked out by the exact method, and comparisons are made with the approximate solutions. H. Lurie, USA

Joints and Joining Methods

(See also Rev. 1465)

1425. Herber, K.-H., Edge joints in tanks and containers (in German), *Stahlbau* **24,** 10, 225–228, Oct. 1955.

Classical small-deformation linear shell theory, together with small-deformation flat plate theory, is applied to the problem of determining the membrane and bending stresses at the juncture of a vertical cylindrical shell and a flat end closure plate. The corresponding problem of a truncated cone closed at one end by a flat circular plate is also investigated. In each of these considerations only axisymmetric conditions of uniform loading and support are considered. Two conditions of support are considered: (a) a reaction distributed uniformly around the boundary of the plate at its juncture with the shell, and (b) an elastic-foundation-type reaction exerted over the entire end closure plate. The results found for conditions (a) present no new information beyond that given by Horvay and Clausen in a recent paper [*J. appl. Mech.* **22,** no. 1, Mar. 1955], although no reference is made to this work. An extremely brief discussion of plastic deformation at the juncture of the shell and its end closure plate is offered. Author indicates that further analysis will be given in a forthcoming paper. W. A. Nash, USA

1426. Stern, E. G., How to build nailed trussed rafters, *Practical Builder,* 50 pp., Nov. 1955.

Complete design, construction, and erection procedures, including stress diagrams, joint details, member dimensions, bills of materials, and specifications, are presented for nailed trussed rafters of 2 in 12 to 6 in 12 roof pitch and 18 to 36-ft. span, with flat or 3 in 12 pitch ceiling. The designs are based on the use of properly helically threaded nails in two- and three-member lumber joints. From author's summary

1427. Marian, J. E., The correlation of 21 variables of the gluing process for wood and similar materials, *Sven. TräforskInst. Medd.* 68B, 10 pp., 1955.

A hypothesis is advanced as to the relative effect of variations in gluing procedures on the effectiveness of a total of 21 gluing variables, which include gluing conditions and glue and wood properties. Numerical data are to be determined experimentally before any such correlation can be applied to laboratory or field conditions. E. G. Stern, USA

1428. Cole, A. G., Johnstone, W. W., and Sheldrick, K. R., Static strength tests on riveted and bolted structural joints, *Aero. Res. Labs., Melbourne, Austral.* SM.219, 11 pp. + 16 figs. Sept. 1955.

Static tests carried out on typical structural joints showed that the stiffness could be increased considerably by using tensioned high-strength bolts in place of mild-steel rivets and that the apparent coefficient of friction of two tension specimens appeared to be in the vicinity of 0.5.

It was concluded that, in general, tightly bolted joints could be used with confidence in place of riveted joints.

From authors' summary

Structures

(See also Revs. 1395, 1399, 1401, 1403, 1415, 1420, 1425, 1426, 1463, 1474, 1475, 1480, 1656, 1674)

1429. Lin, T. Y., Design of prestressed concrete structures, New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd., 1955, ix + 456 pp. $11.50.

Essential feature of this book is comprehensive presentation of all phases of prestressed-concrete structures and knowledge necessary for correct and economical design and construction, with special consideration of American methods and conditions. Sixteen chapters present the information in simple forms on the following subjects: basic principles of prestressed concrete and its development; classification of types, as external, internal, linear, and circular prestressing; pre- and posttensioning; bond of tendons, precast, cast-in-place, and composite construction; stages of loading, safety, and economy; strength requirements of concrete and steel; effect of creep and shrinkage of concrete; manufacturing techniques, use of glass fibers and auxiliary materials; prestressing systems and end anchorages, electrical and chemical prestressing; loss of prestress due to various effects; analysis for flexure, cracking moments, composition sections, arrangement of steel; importance of shear, bond, and bearing; layout and deflections of beams; partial prestress and nonprestressed reinforcement; continuous beams; slabs, single, continuous, and with supporting columns; tension and compression members; circular prestressing, shells, and domes; allowable stresses and load factors; economy and special problems such as fire resistance, fatigue strength, impact, and corrosion resistance; rigid frames. Valuable design data for some prestressing systems used in the USA are added (Freyssinet, Leonhardt, Magnel, Preload, Prescon, Roebling, Stressteel, Texas P.I., Dorland), also constants for various beam sections, and criteria for prestressed-concrete bridges. Special emphasis is given to clarify all problems concerning the design of prestressed-concrete structures by a great number of examples (60). Although elementary knowledge of strength of materials and reinforced concrete is required for understanding this book, it should not be overlooked that good experience in design and construction of prestressed concrete is necessary for better safety and economy.

Author should be commended for inviting the readers, in his preface, to call his attention to any later developments and additional information so that they may be included in future editions. In this connection, one of the first applications of prestressed concrete by K. Wettstein, 1919, should be mentioned, as confirmed by Curzon Dobell and Paul W. Abeles in *ACI Journal,* May and Dec. 1950 [see also *Engineering News Rec.,* Jan. 26, 1950, and *Architect and Engineer,* Feb. 1950]. J. J. Polivka, USA

1430. Fritz, B., Calculation and design of prestressed steel trusses (in German), *Stahlbau* **24,** 8, 169–174, Aug. 1955.

Since the pioneer work of G. Magnel [*Struct. Engr.* **28,** 11, 285–295, Nov. 1950; ibid., **29,** 7, 203–206, July 1951; *Ossature métall.* **18,** 10, 507–519, Oct. 1953] the study of prestressed steel trusses has been a timely problem of steel structural engineering. The present paper marks valuable progress in both the theoretical and the practical aspects of the solution of this problem.

In his cited papers, G. Magnel stipulated for prestressed steel structures that the effect of γ times the service loading should not cause yield stress in the tensile chords. Against this, author requires that, at the same time, yield elongation should not take place either in the prestressing members (Spannglieder). He demonstrates that the arrangement is, in general, the most economical when, under the effect of the γ times loading, both structural members arrive simultaneously at the afore-mentioned

limit state. For economic design it is advisable to position the prestressing members in a way that they mutually overlap each other in the middle part of the truss. Thereby the prestressing force can be adjusted to forces acting in the chords. Such arrangement of the prestressing members calls for the hollow shaping of the cross section of the boom.

Detailed instruction is given on the calculation of trusses prestressed by tensioning members overlapping each other. In contrast to G. Magnel, it is not supposed in the calculation that prestressing is executed stepwise collaterally with the increase of permanent forces, but the problem is treated in its generality. The special case is dealt with when the prestressing members are acting as pulled bars from the beginning toward permanent and non-permanent forces, that is, when they are prestressed by themselves. The solution of the problem is presented when the additional prestressing force is to be applied during loading, or before the total loading has become operative. The case when the prestressing members are tensioned simultaneously on both sides, or when they are tensioned on one side only, is treated in detail.

A numerical example closes the paper. This is also an example of the practical arrangement and fixation of the prestressing members overlapping each other. P. Csonka, Hungary

1431. **Granholm, H., Reinforced timber** (in Swedish, with English and French summaries), *Trans. Chalmers Univ. Technol.* no. 154, 96 pp., 1954.

Timbers reinforced—analogously to reinforced concrete—with flat, square or preferably trapezoidal or triangular steel bars, tightly fitted into milled recessed grooves and firmly glued to the wood, proved to be economical for use in two executed timber structures. In the construction of a 47-ft span foot bridge and a frame for a 50 × 130-ft warehouse, the results of a carefully executed laboratory study were successfully field-tested. The relative lightness of the reinforced beams resulted in expeditious and swift erection of the structures.

Reinforced beams with solid or laminated lumber, plywood, or fiberboard webs were subjected to laboratory tests. Under ideal service conditions, the steel-reinforced construction systems can be successful, as proved by the study. However, especially repetitive dimensional changes of (1) the steel reinforcement with change in temperature and (2) the wood with change in moisture content can, in the reviewer's opinion, result in (a) excessive glueline stresses and (b) undesirable redistribution of internal stresses, which can make the steel reinforcement ineffective and, thus, result in structural failure.

In light of these considerations, the extremely interesting study appears to be of a purely academic nature.

Similarly, however, even more fallacious structural combinations of wood and steel were suggested during 1945 by A. Cibulka in his "Practical design handbook for engineers." The great difference in physical and mechanical properties of the two materials does not allow them to act in unison under most field conditions. E. G. Stern, USA

1432. **Finzi, L., Properties of elastoplastic structures in the "space of hyperstatic structures"** (in Italian), *Acta Pont. Acad. Sci.* **15**, 10, 121–136, 1951.

The determination of states of stress and strain for an elastoplastic hyperstatic truss is made using principles of Haar-von Karman and of Greenberg. The whole work is developed in a space of as many dimensions as there are unknowns (space of hyperstatic structures), using the properties of such space. This space transforms in a plane when the unknowns are only two, and examples are developed for this case.

An interesting work. A. Bignoli, Argentina

1433. **Foulkes, J., Linear programming and structural design,** *Proc. sec. Symp. in Linear Programming*, Washington, D. C., National Bureau of Standards, 1955, 177–184.

Using the principles of plastic design (or limit analysis), author shows that designing a structural frame for minimum material consumption is equivalent to solving a linear-programming problem. When the frame has to support only a single set of loads, this problem is of conventional type; when the frame is to be designed for a multiparameter family of loading conditions, the right-hand side of each subsidiary inequality of the linear programming problem is itself obtained as the solution of an extremum problem. W. Prager, USA

1434. **Onat, E. T., and Haythornthwaite, R. M., The load-carrying capacity of circular plates at large deflection,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-14, 7 pp.

Tests show load-carrying capacity increases with load. Thus, "perfect plasticity" theory predictions in limit analysis are seriously divergent from measured values. Author suggests that discrepancy is due to neglect of strain-hardening and geometrical changes with deflection in limit analysis but he neglects former effect and studies latter. Author's engineering-type approximate analysis for initially flat plate under various loadings and edge effects checks reasonably well with tests. He concludes that yielding of metal in plate is not the most important criterion for loading capacity but rather the rate of deflection change with applied load. Author tentatively suggests possible existence of important theorem that structures adopt shapes most favorable for load-carrying. K. H. Swainger, England

1435. **Broglio, L., Vibrational problems of airplane structures at high speeds** (in Italian), *Aerotecnica* **35**, 4, 171–185, Aug. 1955.

Wing structures are simplified into a simply or doubly symmetrical cantilever box beam of rectangular cross section with stiffeners in the corners and n uniformly distributed stringers in upper and lower panels. These panels are allowed to deform and shear lag is taken into account. Mass is assumed to be concentrated in stiffeners and stringers. Considering only displacements antisymmetric with respect to a horizontal axis of the beam section, this gives $n + 4$ degrees of freedom. Assuming harmonic motion of such structure (bending and torsion of beam), author arrives at a system of $n + 4$ ordinary differential equations for the unknowns as functions of distance along beam axis. For constant section and mass distribution, stringer displacements may be eliminated and solution in closed form obtained. For doubly symmetrical sections, system is further simplified. Bending and torsional modes are then separated and frequency equation may be asymptotically linearized and easily solved graphically. A fully treated numerical example concludes this interesting paper.

F. K. G. Odqvist, Sweden

1436. **Krishnan, S., and Cadambe, V., A note on the minimum weight design of a thin-walled stiffened rectangular cell subjected to torsion,** *J. aero. Soc. India*, **7**, 3, 43–47, Aug. 1955.

Paper derives expressions for width, wall thickness, and stiffener spacing for minimum weight design of the subject cell. Width/depth ratio of section and stiffener size are considered to be fixed, the latter being assumed sufficiently large to localize shear buckling of walls to individual panels. Area of section is minimized subject to conditions that (1) torsional shear stress equals buckling shear stress for individual panels, and (2) unit twist is prescribed. Numerical results for aluminum alloy cells are shown as functions of applied torque and prescribed twist.

L. Maunder, USA

1437. Muckle, W., The influence of proportions on the behaviour of partial superstructures constructed of aluminum alloy, *Quart. Trans. Inst. nav. Arch.* **97,** 3, 453–485, July 1955.

1438. Hirschfeld, K., Some churches with shell-shaped ceilings (in German), *Bauingenieur* **30,** 5, 161–171, May 1955.

There are shown the calculation and the reinforcement of: (1) *A barrel vault shell* (radius 4.335 m = 14.2 ft; thickness 6.5 cms = 2.56 in.; aperture angle 55° 40′). The difficulty of performing the calculation was aggravated by the elastically displaceable support of one edge of the shell and the presence of a lateral shell which perpendicularly penetrates the main shell. Beside the membrane stresses, the bending moments and shearing forces occurring at sections extending longitudinally along the shell were taken into consideration. (2) *Multi-wave shell* comprising ten waves of a wave length of 2.236 m (7.33 ft) and a height of 0.97 m (38.2 in.) each. The stress analysis was based on a special approximation method. (3) *Spherical vault,* comprising a portion of a spherical shell of 21.00-m (68.8 ft) radius and 7-cms (2.76 in.) thickness at the apex, covering an area 22.0 m (72.2 ft) *square.* This vault rests on four corner pillars and is additionally supported by posts, of which six are distributed over the length of each side of the square area covered. There are determined the membrane stresses and the boundary interferences due to shear forces. Calculations are also given for a covering in the form of a diagonal grating and the reinforcement of a circular foundation. G. Sonntag, Germany

1439. Michalos, J., Direct design of two-hinged arches of constant section, *Civ. Engng. N. Y.* **26,** 1, 65–68, Jan. 1956.

Formulas and diagrams are given from which can be found the influence ordinates of the horizontal reaction of two-hinged circular and parabolic arches for different values of the rise-to-span ratio. C. J. Bernhardt, Norway

1440. Markland, E., Properties of circular, parabolic and elliptic arches, *Civ. Engng. Lond.* **50,** 584, 179–180, Feb. 1955.

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 1383, 1387, 1394, 1395, 1396, 1412, 1413, 1432, 1434, 1457, 1481, 1482, 1484, 1577, 1662, 1664)

1441. Marin, J., edited by, Mechanics of creep, *Proc. Short Course, Pennsylvania State Univ.,* June 28–July 1, 1954., 143 pp.

Volume comprises six papers covering various aspects of subject and presented by various authorities at summer short course in 1954 at Penn State University. Material is discussed largely from engineering or design viewpoint, although certain lectures also deal in part with physical and metallurgical aspects of creep problem. Subjects treated include: Theories of creep; fracturing by creep; creep-stress-time relations; mechanics of creep for simple stresses, combined stresses, and members subject to buckling.

Viewpoint of physical metallurgist is represented in paper by J. E. Dorn, who discusses dislocation theory and presents critical review based on experimental data of two widely accepted theories of creep. Experimentally determined laws of high-temperature creep are also reviewed and more rational theory of creep discussed. Material is presented in very readable manner.

Subject of fracturing by creep is discussed by G. R. Irwin in qualitative and descriptive manner. Experiments on bubble raft models and on crack extension in zinc foil specimens are utilized to obtain better picture of process. Some mathematical relations which hold during fracture process are also discussed.

Paper by J. D. Lubahn treats creep properties and creep-stress-time relations of materials from engineering viewpoint. Creep strain is divided into anelastic and plastic components. Plastic behavior at low temperatures, creep at high temperatures, and extrapolation of creep data are discussed. Author points out that mechanical equation of state is often used in spite of its deficiencies, since nothing better is available.

Subject of mechanics of creep and design for simple stresses is treated from an engineering viewpoint by E. A. Davis. This paper discusses various practical problems including: correlation of creep and relaxation test data; design of high-temperature piping; creep in turbine blading; relaxation in bolts for turbine cylinder joints; effect of creep in flanges of such joints.

Problems of creep and design under combined stresses are considered by L. F. Coffin, who discusses primarily the well-known Soderberg and Bailey methods together with modification of the former using maximum shear criterion. Considerable space is given to problem of creep in thick-walled cylinder under internal pressure. It is pointed out that much work, both experimental and theoretical, needs to be done in field of combined stress creep.

Final paper, by F. R. Shanley, treats problems of buckling under inelastic and creep conditions. Mathematical theories of creep buckling and methods for engineering treatment of such problems are discussed; suggestions are also made for further analytical and experimental work.

In reviewer's opinion, volume should be useful to engineers and designers interested in various aspects of creep in machine parts and structures at elevated temperatures since it brings together much important data bearing on the subject. Comprehensive bibliographies given should also be helpful to research engineers and others interested in more advanced aspects of creep problem. A. M. Wahl, USA

1442. Lin, T. H., Creep stresses and deflections of columns, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-43, 5 pp.

Paper gives a direct method for determining stresses and deflections in a pinned end column accompanied by creep. The relaxation moment for a particular time interval is treated as an initial curvature of the column. The method is applicable to stress-strain-time relations obtained directly from test data. An illustrative example is given comparing the results obtained with test data. J. Marin, USA

1443. Weissman, G. F., Pao, V.-H., and Marin, J., Prediction of creep under fluctuating stress and damping from creep under constant stress, *Proc. second U. S. nat. Congr. appl. Mech.,* June 1954; Amer. Soc. mech. Engrs., 1955, 577–584.

Authors present theory of total creep composed of three parts: one elastic, one viscous, and one transient, each of which follows its own laws. Thus the first is assumed to follow Hooke's law and the second Norton's power law for minimum creep rate in secondary creep. The third part is assumed to be recoverable and the transient creep rate to be proportional to the difference between a power function of stress and the transient creep itself. In this way, for any given time-dependent stress, total creep may be computed by quadratures. Formulas for sinusoidally varying stress are worked out in detail for special cases. Experiments with Plexiglas in a dynamic creep testing machine, described elsewhere [AMR **8,** Rev. 3078], are stated to fit said formulas with reasonable accuracy.

Reviewer doubts the possibility of applying theory to gas turbines and jet engines as stated by authors in introduction, reason being that irrecoverable primary creep may play a dominant part.

The work done in one stress cycle is calculated as based upon

above theory and favorably compared with experiments on damping capacity by J. A. Sauer and W. J. Oliphant [AMR **4**, Rev. 1603]. Unfortunately, it is impossible to check authors' calculations, as table 4.1, referred to in the text, is missing.

F. K. G. Odqvist, Sweden

1444. **Mandel, J., The flow of a Maxwell body** (in French), *C. R. Acad. Sci. Paris* **241**, 22, 1550–1552, Nov. 1955.

Analysis of the special case when stresses are instantaneously imposed at time zero and thereafter remain constant. General solution includes series of relaxation terms with discrete distribution of relaxation times. Application is made to a semi-infinite body with concentrated normal force acting on its face.

M. Mooney, USA

1445. **Garr, L., Lee, E. H., and Wang, A. J., The pattern of plastic deformation in a deeply notched bar with semicircular roots,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-23, 3 pp.

On the basis of the theoretical paper of A. J. Wang, authors develop a method to make clear the plastic flow in a deeply notched bar with semicircular roots pulled in tension. The deformation of a square grid scribed in a cross section of the undeformed bar demonstrate, step by step, the plastic flow. Large strains occur in the transition region between the initial and final positions of the plastic-rigid boundary and also adjacent to the notch roots. It is hoped that the computed patterns will be of value in assessing the initiation of fracture in a notched bar test.

L. Föppl, Germany

1446. **Huth, J. H., A note on plastic torsion,** *J. appl. Mech.* 22, 3, 432–435, Sept. 1955.

Author deals with the uniform torsion of noncircular rods, comparing flow theory of W. Prager with general deformation theory of Nadai. Starting from differential equation of Prager, he presents, for a chrome-nickel steel rod with square section, the solution in finite-difference approximation. Since shear modulus depends on shear stresses, it is necessary to repeat procedure many times. Convergence is checked by halving intervals for finite-difference process. Then equation of Nadai is solved by similar procedure.

Author presents diagrams of effective torsional rigidity and maximum shearing stresses as a function of angle of twist, obtaining only slight differences between theory of Prager and Nadai.

H. Beer, Austria

1447. **Davis, E. A., Combined tension-torsion tests with fixed principal directions,** *J. appl. Mech.* **22**, 3, 411–415, Sept. 1955.

Series of tests are conducted on cylindrical tubes under conditions indicated in title with internal pressure added. Results show that, in these tests, finite strains with fixed principal directions can be obtained from the summation of the increments along a fixed direction; also, that the octahedral shearing stress is a unique function of the octahedral shearing strain and the principal directions of stress and strain coincide.

J. P. Vidosic, USA

1448. **Biot, M. A., Deformation of viscoelastic plates derived from thermodynamics,** *Phys. Rev.* **98**, 6, 1869–1870, June 1955.

Differential equations are set up by which the displacements are determined as functions of the coordinates and the time. In some degree of approximation the equations are of the same form as the corresponding equations for elastic plates, time operators being substituted for constants. The reference in the title to thermodynamics means that certain approximations (conventional in irreversible thermodynamics) are made with respect to the rate of dissipation. No quantitative results are deduced from the differential equations; it is not stated to which materials the equations are applicable. For these reasons the use of the present note is limited.

R. Eisenschitz, England

1449. **Jung, H., On the application of Hill's minimum condition in the theory of plasticity** (in German), *Ing.-Arch.* **23**, 1, 61–68, 1955.

Approximate solution of stresses in closed thick tube under uniform interior pressure is presented for strain-hardening material. Using minimum condition, approximate solution is improved and briefly compared with test results. Reviewer considers comparison unconvincing since neither final theoretical relationships nor experimental results are given explicitly.

G. G. Meyerhof, Canada

1450. **Watts, A. B., and Ford, H., On the basic yield stress curve for a metal,** *Instn. mech. Engrs. Proc.* Preprint, 11 pp., 1955.

Basic yield stress curve in plane strain is derived by compression between smooth parallel dies, technique being applied experimentally to high-conductivity copper strips. Cold-rolling process is investigated; anomalies are found in rolling thick material. Results depend on small differences in behavior and, although authors give some measures of experimental error, fuller details would be helpful in interpreting results.

J. Heyman, England

1451. **Serensen, S. V., and Meshchaninova, G. P., Tensometric determination of residual stresses in disks of varying thickness** (in Russian), *Zh. tekh. Fiz.* **24**, 3, 473–478, Mar. 1954.

1452. **Moroz, L. S., Significance of the characteristics of plasticity representing various physical states of alloys** (in Russian), *Zh. tekh. Fiz.* **24**, 3, 425–432, Mar. 1954.

1453. **Sentis, A., Note on a theory of viscoelasticity of materials** (in French), *C. R. Acad. Sci. Paris* **241**, 24, 1700–1702, Dec. 1955.

Paper examines a particular solution of partial differential equation $au_{xxt} + u_{xx} - bu_{tt} = 0$ and shows that it can be interpreted as propagation of decaying plane wave or decaying of stationary wave.

G. W. Housner, USA

1454. **Cole, J. A., Petersen, R. E., and Emmons, H. W., Experiments with a rotating-cylinder viscometer at high shear rates,** *NACA TN* 3382, 31 pp., June 1955.

Paper is the fifth report in a series describing the work undertaken by H. W. Emmons and his associates on the viscosity of lubricating oils and should be read in conjunction with the first four reports [Emmons, H. W., Head, V. H., Batteau, D. W., and Hammond, J. M., "A high rate of shear viscosimeter," Contract NAw-5630, NACA and Harvard Univ., Aug. 12, 1949; Emmons, H. W., and Head, V. H., "Preliminary tests of a rotating viscosimeter," Contract NAw-5799, NACA and Harvard Univ., Nov. 1950; Emmons, H. W., and Soo, S-L., "Development and preliminary tests of a rotating viscosimeter," Contract NAw-6043, NACA and Harvard Univ., May 1952; Cole, J. A., NACA Lubrication Project Research Report, Contract NAw-6043, NACA and Harvard Univ., Mar. 1953.]

Paper describes tests on two mineral oils and an oil containing a polymer additive at high rates of shear inclusive of accom-

panying heat effects. The torque measurements are of low accuracy and fail to establish the moderately non-Newtonian behavior of the polymer-containing oil, but the temperature measurements, which are in good agreement with a thermal analysis, (given in an appendix) do indicate the presence of temporary viscosity decrease at high shear rates for this oil.

Paper contains an important contribution to understanding of thermal effects in lubrication of high-speed bearings.

J. Kestin, USA

1455. Becker, F., and Schramli, W., The flow properties of non-Newtonian fluids (in German), *Zement-Kalk-Gips* **8,** 10, 345–352, Oct. 1955.

Failure, Mechanics of Solid State

(See also Revs. 1423, 1470, 1478, 1621, 1645, 1670)

1456. Mathauser, E. E., and Deviekis, W. D., Investigation of the compressive strength and creep lifetime of 2024-T3 aluminum-alloy plates at elevated temperatures, *NACA TN* 3552, 40 pp., Jan. 1956.

The results of elevated-temperature compressive strength and creep tests of 2024-T3 (formerly 24S-T3) aluminum-alloy plates supported in V-grooves are presented. The strength-test results indicate that a relation previously developed for predicting plate compressive strength for plates of all materials at room temperature is also satisfactory for determining elevated-temperature strength. Creep-lifetime results are presented for plates in the form of master creep-lifetime curves by using a time-temperature parameter that is convenient for summarizing tensile creep-rupture data. A comparison is made between tensile and compressive creep lifetime for the plates, and a method that makes use of time-dependent stress-strain curves for predicting plate-creep failure stresses is investigated.

From authors' summary

1457. Garofalo, F., Smith, G. V., and Royle, B. W., Validity of time-compensated temperature parameters for correlating creep and creep rupture data, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-164, 11 pp. + 8 figs.

Several time-compensated temperature parameters, which were proposed as dependent only on the initial stress, are discussed with relation to time of creep rupture and to minimum and average creep rates. The so-called master "curve" is discussed in terms of experimentally determined relationships. In checking the validity of the proposed parameters, using experimental results for several ferritic and austenitic steels, it is found that these parameters are not single-valued functions of the initial stress over wide ranges in stress, and the existence of a master curve for these parameters is doubtful.

From authors' summary

1458. Sinclair, G. M., Corten, H. T., and Dolan, T. J., Effect of surface finish on the fatigue strength of titanium alloys RC 130B and Ti 140A, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-197, 13 pp. + 3 tables, 9 figs.

An investigation was made of the effect of various surface-finishing operations on the fatigue strength of two titanium alloys, RC 130B and Ti 140A. The types of finish studied included rough-machined, machined and mechanically polished, cold-rolled, electro-polished, and ground surfaces. Through the use of microhardness measuring techniques it was found that the different finishing methods introduced varying degrees and varying depths of cold work in the surface layers of the metal. Cold-rolling produced the highest hardness in the surface layer, while grinding gave the lowest value; in one case the ground surface appeared to be slightly softer than the inner core metal. In general, the fatigue strength for lifetimes exceeding 2×10^7 cycles was found to vary according to the hardness of the surface layer with the highest hardness corresponding to the greatest fatigue strength. Roughness of the surface was also found to influence fatigue strength but to a much lesser degree than hardness. As a first approximation, the relationship between surface hardness, surface roughness, and fatigue strength could be expressed by an equation of the form $Z = KX^{-a}Y^{b}$, where Z is fatigue limit, X surface roughness, Y hardness, and K, a, and b are constants of the material. Data from the present study were evaluated in terms of this equation and the results are presented in the form of a nomograph. Results of tests made on notched specimens appear to indicate that titanium is less notch sensitive in fatigue than was previously reported. Early reports of extreme notch sensitivity may have resulted from comparisons made between notched and smooth specimens having quite different surface preparations in the test section.

From authors' summary

1459. Schabtach, C., Rankin, A. W., Fogleman, E. L., and Winne, D. H., Report of the investigation of two generator rotor fractures, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-208, 26 pp.

1460. Rankin, A. W., and Seguin, B. R., Report of the investigation of the turbine wheel fracture at Tanners Creek, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-210, 27 pp.

1461. Sines, G., Failure of materials under combined repeated stresses with superimposed static stresses, *NACA TN* 3495, 69 pp., Nov. 1955.

A review of experiments on biaxial alternating stresses and simple combinations of static stress with alternating stress is shown to lead to a general criterion for the effect of static stress on the permissible amplitude of alternating stress. The proposed criterion is then shown to agree with results from tests that have been performed under more complex stress states. Tests were performed to determine the effect of static compression on alternating torsion, the one simple combination that had not been previously investigated. These results also agree with the criterion. It is shown that Orowan's theory of fatigue can be modified to predict the observed effect of mean stress on the permissible amplitude of alternating stress.

From author's summary by R. Hill, England

1462. Findley, W. N., Mitchell, W. I., and Strohbeck, D. D., Effect of range of stress in combined bending and torsion fatigue tests of 25S-T6 aluminum alloy, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-68, 7 pp.

Authors present fatigue test data obtained by means of repeated-deflection type of fatigue-testing equipment. The results were corrected for effects of nonlinear stress distribution and stress relaxation. The influence of anisotropy, mechanism of crack formation, mean stress, and maximum stress is discussed. One of authors' conclusions is that completely reversed stress cycles above the proportional limit may have increased the fatigue resistance.

A. Yorgiadis, USA

1463. Legett, J. L., Investigation of fatigue strength of railroad timber bridge stringers, *Bull. Amer. Rly. Engng. Assn.* **55,** 510, 161–211, Sept./Oct. 1953.

Test data are presented on the fatigue behavior of green, non-treated southern pine and Douglas fir small-size standard-test specimens and one-quarter and full-scale stringers, such as are in common use in railway bridges and trestles.

Timber checks increased during drying during fatigue testing, caused sudden horizontal-shear failure which originated generally at the specimen end. Fatigue failures in bending green, clear, small-size specimens started with compression failure near the point of load application. Notched green, clear specimens failed in bending and notched dry specimens failed in horizontal shear.

At fiber stresses in bending amounting to 50% of the modulus of rupture, over 30 million stress cycles can be endured by clear, green, straight-grained specimens and, at repeated fiber stresses up to 60%, over 20 million cycles can be sustained.

E. G. Stern, USA

1464. Fukui, S., and Miki, S., Change of strain duing torsional fatigue for special steels for constructional use (in Japanese), *Rep. Inst. Sci. Technol. Tokyo* **9,** 1, 31–39, Mar. 1955.

Change of strain amplitude, maximum strain, and residual strain during torsional fatigue by Ni-Cr-Mo and Cr-Mo steels for constructional use are shown in this paper. Results for both steels by mean stress of $0 \sim 40$ kg/mm^2 are as follows: Strain amplitude and maximum strain increase from about 10^4 reversal of stress for fatigue limit and reach to maximum at about $3 \sim 5 \times 10^5$, then settle to certain values. They increase monotonously when the specimens break down. Amounts of both changes are larger for Cr-Mo steel than for Ni-Cr-Mo steel. Results for Ni-Cr-Mo steel having residual strain by prestressing of $60 \sim 80$ kg/mm^2 are as follows: Torsional fatigue limit decreases a little for prestress up to about 70 kg/mm^2. Strain amplitude and maximum strain decrease slightly from about 10^4 reversal of stress for fatigue limit, but the amount of change is small. Preworked residual strain decreases gradually and residual stress is thought to decrease also. From authors' summary

1465. Harris, L. A., Nordmark, G. E., and Newmark, N. M., Fatigue strength of butt welds in structural steels, *Welding J.* **34,** 2, 83s–96s, Feb. 1955.

Fatigue tests have been conducted of full-scale butt welds in carbon-steel plates with the stress applied either parallel to or perpendicular to the direction of welding. The welds were deposited with either the E7016 low-hydrogen electrode or the E6010 high-cellulose electrode. The specimens were tested with either the reinforcement on or removed by grinding.

For the specimens tested with the stress perpendicular to the direction of welding, there was no significant difference in the fatigue strength of welds produced with the two types of electrode and tested with the reinforcement on or off. For the specimens tested with the stress parallel to the direction of welding, the scatter is large, but the E7016 welds with the reinforcement either on or off are generally more resistant to repeated loadings than the E6010 welds, especially if failure occurs after a relatively low number of highly stressed cycles.

An extensive re-evaluation of the past fatigue tests of butt welds conducted at the University of Illinois has been made to evaluate the use of different welding procedures.

From authors' summary

1466. Fitchie, J. W., Some experiments into the behaviour of plain and notched steel specimens under static and fatigue loadings, *Instn. mech. Engrs.* Preprint, 15 pp., 1955.

The object of the work was to determine the true stress field at the base of a notch in specimens when loaded within and beyond the elastic range. With this information, fatigue tests were then designed with the objective of correlating results from plain specimens with those from notched specimens for fatigue cycles with and without a superimposed mean stress.

From author's summary

1467. Findley, W. N., Mitchell, W. I., and Martin, D. E., Combined bending and torsion fatigue tests of 25S-T aluminum alloy, *Proc. second U. S. nat. Congr. appl. Mech.,* June 1954; Amer. Soc. mech. Engrs., 1955, 585–593.

Tests at zero mean stress in bending torsion and three combinations thereof are described, reference being made to previous papers for description of machines and testing technique. Specimens were considered to have failed when stiffness was reduced to $^7/_8$ of its original value.

Results are given in double-logarithmic plots of alternating principal shear stress versus cycles to failure and can be approximated by parallel straight lines up to stresses well above proportional limit. Change of stress-strain curve during first cycles of loading is given, showing approach to linear relation after a few cycles. Fatigue test results are compared with six theories of failure, the best agreement being obtained with "principal shear stress corrected for anisotropy." F. J. Plantema, Holland

1468. Templin, R. L., Fatigue of aluminum, *Proc. ASTM* **54,** 641–699, 1954.

Minute fissures are produced along slip planes in aluminum repeatedly stressed at specific stress level. These fissures progressively join with other fissures on the same planes, with neighboring planes, and with other fissures in adjacent crystals to form a continuous fatigue crack. Axial fatigue stresses are lower than flexural or stresses obtained with rotating beam. Aluminum withstands considerably higher compressive than tensile fatigue stresses. High purity aluminum fatigue specimens should preferably be machined from severely cold-worked material and then annealed. Speed of testing seems not to have significant effect on fatigue values. Testing technique and equipment affect fatigue results. From a scatter band, which is considerable in aluminum, a mean curve can be established. Fatigue data obtained with smooth specimens and from rotating beam tests are the most consistent. Chemical composition, tempera, working, or ductility have little effect on endurance limit of severely notched specimens whether obtained with cast or wrought alloys. Solution heat treatment increases fatigue strength of smooth and notched specimen but precipitation heat-treatment does not, and may even lower it. Cold reduction after heat-treatment, grain size, and grain direction seem not to have significant effect on fatigue. Intermetallic compound particles which alloy with aluminum do not have deleterious effect, but those which do not alloy may have; internal seams, voids, intergranular corrosion, and unhealed porosity are harmful—especially for smooth specimens. Compressive surface stresses obtained by shot peening, cold surface rolling, or heat-treatment improve fatigue strength. Preloading in tension may increase the notch endurance limit by as much as 100%, while preloading in compression may decrease it by 50%. The temperature below room increases, and above room decreases smooth specimens' fatigue strength. At about 600 F all commercial aluminum alloys and forms have practically the same endurance limit. Heavy oxide coating, cladding, corrosion, tool marks, nicks, etc., decrease fatigue properties. Stress raisers are the primary causes of fatigue failures. Poor agreement exists between the geometric stress-concentration factors and experimental fatigue-strength reduction factors. Results from laboratory tests on fatigue specimens give useful information but cannot directly be applied to design. Data from service performance are the best, while simulated fatigue tests on components and assemblies are the next best. Reasonable agreement between component and assembly data has been found. Stress raisers are the most important factors above 10^6 cycles, and static strength of material seems to have no effect on fatigue strength of components above that cycle number. Double strap butt riveted joint has about twice the fatigue strength of

simple lap joint with a single row of rivets. Static preloading of a riveted joint with mismatched rivet holes improves joint fatigue. 497 references. J. Koziarski, USA

Material Test Techniques

(See Revs. 1456, 1642)

Mechanical Properties of Specific Materials

(See also Revs. 1403, 1427, 1431, 1437, 1450, 1452, 1453, 1458, 1462, 1464, 1467, 1468, 1598, 1640, 1669)

1469. **Yarwood, J., High vacuum technique. Theory, practice, industrial applications and properties of materials, 3rd ed.,** New York, John Wiley and Sons, Inc., 1955, 208 pp. $5.50.

This third edition preserves the purpose of the former editions to present a survey of current equipment, practice, and materials; but the text has been revised and extended. The treatment is not intended to be basic or complete. The reader is sometimes warned of the intrinsic experimental difficulties as indicated by this example from p. 149. After a brief description of techniques to obtain good photocells, author says: "Only by several months' preparatory experimental work can first-class results be obtained."

This book provides an excellent survey of vacuum pumping equipment, vacuum measuring technique, out-gassing, and "gettering," with an abbreviated list of industrial applications and properties of materials for vacuum techniques. Over a third of the book is devoted to particularly complete descriptions and comparisons of the design and rating of mechanical forepumps with and without gas ballast to remove condensable vapors, of diffusion and of ejector pumps of British, American, and German manufacture. With this guide, a novice to vacuum techniques should be able to select the best equipment for a specific application and be forewarned of the major difficulties.

The style of writing is clear, although the descriptions are very short, by policy of the author. There are very few errors and the index is very complete, which is most desirable for a small reference book of this type. F. Todd, USA

1470. **Gemmill, M. G., and Murray, J. D., Bolting materials for high-temperature steam-plant service,** *Engineering* **180,** 4690, 824–827, Dec. 1955.

1471. **Sully, A. H., High temperature coatings for metals,** *Prod. Engng. (Product Design Handbook for 1956)* **26,** 11, C8-C11, Oct. 1955.

1472. **Foley, F. B., Mechanical properties at elevated temperatures of ductile cast-iron,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-204, 6 pp. + 5 figs.

The elevated temperature properties of ferritic ductile cast iron for 100-deg intervals from 700 to 1200 F are summarized in a table in which the values, other than those for short-time tensile tests, have been computed by means of the Larson-Miller equation. These are quite close approximations of values to be expected from wholly ferritic ductile iron. The presence of combined carbon will tend to increase these values particularly in the low temperature range, so that from this point of view they are minimum values. From author's summary

1473. **Anonymous, Engineering survey on ductile iron,** *Prod. Engng. (Product Design Handbook for 1956)* **26,** 11, B2-B7, Oct. 1955.

1474. **Pall, D. B., Porous metals in aircraft,** *Aero. Engng. Rev.* **13,** 7, 36–41, July 1954.

1475. **Norton, J. T., The development of cermets as structural materials,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-196, 9 pp. + 8 figs.

Cermets have been developed in an attempt to provide materials which would permit operating temperatures in devices such as turbojet engines, gas turbines, rocket motors, and certain parts of nuclear reactors to be raised beyond the limits imposed by the conventional high-temperature alloys which now lie in the range between 1500 and 1650 F. There have been available for a long time high melting substances such as the refractory ceramic materials which retain their strengths at temperatures considerably above those mentioned, but they were generally unsatisfactory for the type of application cited because of their fragility; that is, their low resistance to both thermal and mechanical shock. However, there were many reasons for believing that by suitably combining the ceramic materials with metals, bodies could be obtained in which the ceramic would supply the backbone necessary to carry the loads at high temperature while the metal component would contribute a measure of toughness sufficient to permit practical application. Thus the cermets were born. From author's summary

1476. **Rea, R. F., Ceramics for mechanical applications,** *Prod. Engng. (Product Design Handbook for 1956)* **26,** 11, C5–C7, Oct. 1955.

1477. **Stavrolakis, J. A., Cermets for thermal shock,** *Prod. Engng. (Product Design Handbook for* 1956) **26,** 11, B22–B24, Oct. 1955.

1478. **Findley, W. N., Peithman, H. W., and Worley, W. J., Influence of temperature on creep, stress-rupture, and static properties of melamine-resin and silicone-resin glass-fabric laminates,** *NACA TN* 3414, 71 pp., Jan. 1956.

Results of the following tests of melamine-resin glass-fabric laminates and silicone-resin glass-fabric laminates at temperatures up to 400 and 600 F, respectively, are reported: Static-tension, static-compression, tension-creep, and time-to-fracture tests.

The mechanical properties of both laminates weakened with increase in temperature, as a rule.

The creep data supply additional evidence that the per cent increase in strain from one given time to another given time (called "creepocity") is independent of stress. An analysis of the creep data is represented by an equation which describes the effect of stress, time, and temperature. This equation is based on the activation-energy theory and a power function of time.

An analysis of the data shows the possibility that no creep will occur at temperatures below −10 F for the melamine-resin laminate and −119 F for the silicone-resin laminate.

From authors' summary

1479. **Kuehne, H., The influence of moisture content, specific gravity, direction of grain, and angle of annual rings on strength and deformation of Swiss spruce, fir, larch, red beech, and oak,** Rep. no. 183, Federal Materials Testing Laboratory for Industry, Construction, and Trades, Zurich, 122 pp., Feb. 1955.

The results of a systematic and intensive study of the properties of 34 trees from five Swiss wood species are reported on by means of numerous photographs and diagrams, which graphically present approximately 50,000 basic test data obtained during 1937–1942 and 1947–1954. The test results should form a basis for better utilization of the wood species investigated.

E. G. Stern, USA

1480. **Wille, F., Reflections on modern wood construction** (in German), *Holz als Roh und Werkstoff* **13,** 1, 25–32, Jan. 1955.

Based on his experience in the field of wood construction, author suggests the performance of research on the influence of structural deformation due to internal forces on stresses occurring in timber structures. These internal forces result from eccentric arrangement of structural members at the joints, single and repetitive changes in moisture content of the lumber, creep and plastic deformation of locally overstressed wood.

The most successful framing systems in use are illustrated at hand of executed designs. Suspended framing systems, with catenary steel suspension wires within the framed planes of the structure, are suggested by the author for further study.

E. G. Stern, USA

Mechanics of Forming and Cutting

1481. **Frisch, J., and Thomsen, E. G., Residual stresses in cold extruded aluminum,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-27, 4 pp.

A 1.5-in.-diam cylindrical bar was extruded at room temperature from an 1100-0 (2S-0) aluminum 4.3-in.-diam billet. Residual compressive stresses in the longitudinal direction were found to exist on the surface of the extruded bar. These stresses, as determined from the axially relaxed stresses in the billet during extruding, were found to be in qualitative agreement with the experimentally determined residual stresses as found by Sachs' boring-out method. The severeness of work-hardening at the edge of the die, found from the hardness pattern on the sectioned partially extruded billet, was in close agreement with the distribution of natural strains obtained under the same conditions of extrusion. From authors' summary by H. A. Lepper, Jr., USA

1482. **Sedlaczek, H., Hydrodynamic theory of rolling** (in German), *Stahl u. Eisen* **75,** 9, 593–596, May 1955.

A. Kneschke [*Bergakademie* **6,** 1–11, 1954] has formulated a theory according to the viscous fluid laws by including the best accepted theories about the rolling process in the roll gap as proposed by Th. von Karman, E. Siebel, A. Geleji, and others, all of whom start with a static rolling process by considering an empirical analytical form of the resistance to deformation. For the investigation of what occurs in the gap between the rolls, the equations for the flow of viscous fluids by G. G. Stokes are applied by including the limits of the gap between the rolls as a parabolic arc whose parameter corresponds to the diameter of the rolls. A. O. Schmidt, USA

1483. **Gardiner, F. J., The springback of metals,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-66, 7 pp.

Springback in sheet metal forming can be a troublesome phenomenon. Much experimental and some theoretical work has been performed to date. This paper treats the two-dimensional problem by assuming the desired curve to be a combination of circular arcs prior to and after springback. Using the customary flexure-theory assumption that the strain in a fiber is directly proportional to its distance from the neutral axis and a simplified stress-strain diagram, a relation involving the radii of curvature before and after springback, along with mechanical properties, is derived. Experimental checks using several metals reveal good agreement between theoretical and experimental values. Corrections are still necessary to account for local and end conditions. Author claims formulation can be applied to materials other than those tested. J. P. Vidsic, USA

1484. **Alexander, J. M., A slip line field for the hot rolling process,** *Instn. mech. Engrs.* Preprint, 8 pp.

A slip-line field is constructed to simulate the conditions of plastic flow in hot strip rolling. The method of construction is based on the geometrical representation proposed by Prager, which consists of the slip-line field in the physical plane, loci (cycloids) of stress poles in the stress plane, and the velocity image in the hodograph plane. The assumptions used, in addition to those inherent to an ideally plastic material under plane strain, are (1) that the shear stress along the contact between the roll and the strip is equal to the yield strength of the material in pure shear, or "sticking friction" prevails everywhere; (2) that the roll contour at the contact is a circular arc which is tangent to the strip leaving the roll gap, or that the roll deformation is negligible.

By trial and error, author constructs a possible slip-line field which satisfies stress and velocity conditions at the boundary for one specific rolling geometry having a reduction of 0.332 and a roll-diameter-to-strip thickness ratio of 116.

The slip-line field obtained resembles qualitatively that of compression of strips between rough platens except that (1) there is no singularity at the exit side and (2) there is an additional thin zone of dead metal at the entering side of the roll gap, where the velocity discontinuity would explain the sticking of the material to the rolls observed sometimes under certain conditions.

Since the problem involves rotation of the boundaries, discussion of the effect of such rotation on the hodograph is also included in the paper. K. N. Tong, USA

1485. **Horne, M. R., The elastic-plastic theory of containers and liners for extrusion presses,** *Instn. mech. Engrs. Proc.* **169,** 4, 107–122, 1955.

See AMR **8,** Rev. 372

1486. **Letner, H. R., Influence of grinding fluids upon residual stresses in hardened steel,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-123, 6 pp. + 4 figs.

Grinding fluids perform functions which might be expected to affect the residual surface stresses resulting from a grinding operation. Stresses were analyzed in bars of hardened ball-bearing steel, surface ground in the presence of thirteen different fluids. The results suggest that the effectiveness of a fluid in minimizing residual grinding stresses depends not so much upon its capacity for removing heat from the surface as upon its ability to reduce frictional forces between the wheel and the work.

From author's summary

1487. **Niedzwiedzki, A., Lubrication in the cutting of metals, Part I,** *Machinery Lond.* **83,** 717, 1–6, Oct. 1953; part II, **84** 337, 1–7, Feb. 1954; part III, **85,** 280, 1–7, Aug. 1954; part IV, **85,** 1241, 1–9, Dec. 1954; Part V, **85,** p. 1295, Dec. 1954.

1488. **Wunderlich, W., Graphical and numerical methods to determine the form of helical grooves milled with a given cutter. II** (in German), *Ost. Ing.-Arch.* **9,** 4, 273–280, Nov. 1955.

See AMR **6,** Rev. 1933.

1489. **Sutherland, J., Foundry automation and the shell-molding process,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-162, 5 pp. + 17 figs.

The shell-molding process represents to the foundry industry a step closer to the automatic or push-button foundry. The process, which consists of making thin resin-bonded shells, provides comparatively light molds and a subsequent reduction in

material requirements, simplifying the design of automatic equipment. The foundry industry, through mechanization, has been gradually approaching automation during the past 20 or 25 years, and the shell-molding process will greatly advance this degree of automation. From author's summary

1490. Jun, F. G., and Gehrig, E. J., Pneumatic conveying as adapted to the milling industry, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-166, 8 pp.

Hydraulics; Cavitation; Transport

(See also Revs. 1361, 1506, 1529)

1491. Lin, C. C., The theory of hydrodynamic stability, New York, Cambridge University Press, 1955, xi + 155 pp. $4.25.

The scope of the volume may best be given by quoting from the preface: "The present volume is limited mainly to the study of the stability of motion of a homogeneous viscous fluid, with respect to infinitesimal disturbances, that is, the natural modes of small oscillations of such a mechanical system. Many other interesting problems, such as the stability of the interface between two different fluids, are therefore left out. . . .Two distinct prototypes of instability are presented by the two simplest types of flow: namely, the Couette flow, . . . and the Poiseuille flow, . . . A reasonably detailed treatment of these two cases will therefore form the central part of the theoretical development. . . .The case of the boundary layer will also be given extensive treatment. . . .The problem of transition to turbulence . . . will not be discussed in detail in the present treatment. . . ."

The introduction treats the problem of hydrodynamic stability, its mathematical formulation for an incompressible fluid, and also includes some examples and general remarks. In the second chapter, the stability of Couette motion is discussed by giving an approximate treatment and then the analysis due to G. I. Taylor. The third chapter treats the stability of plane Poiseuille flows by discussing two- and three-dimensional disturbances, giving the sufficient conditions for stability, and by giving two methods (the second is an improved one) of asymptotic solutions. The fourth chapter deals with the general theory of hydrodynamic stability, the fifth with the boundary layer over a flat plate, the sixth with other nearly parallel flows, and the seventh with examples of stability problems in astrophysics and geophysics. The last chapter gives the mathematical theory for the stability of parallel flows.

The book is written with the well-known thoroughness of the author and is a welcome contribution to the literature on hydrodynamic stability. An impressive list of references is given.
T. P. Torda, USA

1492. Bonnemoy, G., Equilibrium profile of rivers with mobile beds (in French), *Ann. Ponts Chauss.* **125,** 3, 284–322, May/June 1955.

This study brings, first of all, some complements to Bachet's method in the study of flood propagation, so as to take into account the variedness of the slope.

The obtained data, which are valid for the variations of the surface, are then extended through the same method to those of the bed of the river.

This extension allows approaching the research on the form of the equilibrium of the longitudinal section of rivers having a mobile bed, when certain conditions are fulfilled.

The form of the equilibrium is immaterial if the grading of the constitutive material of the bed is, in function of the slopes and the rivers hydraulic regime, adequately chosen.

Thus any section can be a section of equilibrium, but these equilibria are unevenly stable. The form of the most stable section is established by means of the calculation of the variations. It is correctly verified by the real longitudinal section of the Tech River, located between Céret and the sea, which, moreover, fulfilled the conditions which had served as a basis for the theoretical study. From author's summary

1493. Bouvard, M., Molbert, J., and Gerard, P., A discussion of expansion chamber-type surge tanks (in French), *Houille blanche* **10,** 4, 506–521, Aug.-Sept. 1955.

First, the authors study the possibilities of improving—by means of a very simple device—the damping of oscillations in a surge tank with lower and upper expansion chamber. Then they propose a particular case, a type of surge tank in which the damping of oscillations is obtained for half a period.

The method of calculation used by the authors is the classical method they have previously proposed.

In the particular case they mention (head-loss, low head, and small variation of the static level) and whatever operations are performed, damping of oscillations is very quick, which allows the execution of simultaneous maneuvers without special precautionary measures.

These advantages are of great interest in the case of a power peak station. L. Escande, France

1494. Moons, P., Application of calculation of oscillations of a mass of water in a surge tank (in French), *C. R. Acad. Sci. Paris* **240,** 2, 154–155, Jan. 1955.

1495. Slepoĭ, Ju. Sh., Testing of efficiency of dredge pipes in field conditions (in Russian), *Gidrotekh. i Melior.* **7,** 7, 29–32, July 1955.

A short nozzle with an angle of 13°24′ is attached to an outlet; head is observed in a piezometric tube. Correction is necessary for the specific weight of pulp S. Kolupaila, USA

1496. Wislicenus, G. F., Critical considerations on cavitation limits of centrifugal and axial-flow pumps, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-144, 10 pp.

This interesting paper starts with the proof that, comparing theoretical and laboratory test results of airfoil cascades with accepted cavitation inception figures of centrifugal and axial-flow pumps, these machines must cavitate at much lower suction specific speeds, but evidently the effects are small enough to be disregarded in practice.

Author calls attention to the beneficial effect of the bound vortex lines of the vanes, which thus far have not been investigated. He shows, like other authors, that prerotation to a certain optimum degree permits a higher suction specific speed for incipient cavitation.

After giving a rational over-all picture of the flow in inducers [see Ross, C. C., and Banerian, G., Rev. 1568 in this issue] which operate with fully developed cavitation, author proposes a novel daring pump-impeller design of the impulse type, similar to a Laval steam-turbine rotor, operating at high heads with fully developed cavitation at very high suction specific speeds (5000 ft head, about 75,000 suction specific speed in the example). As the full head plus friction and velocity transformation losses appear at the outlet of the impeller as velocity head, reviewer fears that author's pump efficiency estimate (70–80%) is too optimistic. A. Hollander, USA

1497. **Markland, E., and Pope, J. A., Experiments on a small pump suction well, with particular reference to vortex formations,** *Instn. Mech. Engrs.* Preprint, 13 pp.

These experiments were carried through at the University of Nottingham with a 2.58-in. inside diam suction pipe, two different suction bells, and various inlets. Some sump configuration with or without obstructions within them complement Denny's data. A. Hollander, USA

1498. **Edge, P. M., Jr., Hydrodynamic impact loads in smooth water for a prismatic float having an angle of dead rise of 10°,** *NACA TN* 3608, 20 pp., Jan. 1956.

Hydrodynamic-loads data obtained at the Langley impact basin from tests of a prismatic float with an angle of dead rise of 10° are presented. Tests were made at fixed trims of 3°, 12°, and 30° and a range of flight-path angles from 2° to 23°. The data are presented as dimensionless coefficients, and experimental values of dead-rise function and aspect-ratio factor are obtained. From author's summary

Incompressible Flow: Laminar; Viscous

(See also Revs. 1381, 1491, 1494, 1524, 1536, 1542, 1544, 1546, 1547, 1554, 1568, 1569, 1570, 1575, 1585, 1616, 1622, 1653, 1660)

1499. **Slezkin, N. A., Dynamics of viscous incompressible fluid [Dinamika vjazkoi neszhimaemoi zhidkosti],** Moscow, Gos. Izdat. Tekh.-Teor. Lit., 1955, 519 pp. 10 r. 90 k.

This attractive work originates in the author's lectures at the mechanicomathematical faculty of the Moscow University. It is written as a textbook for the students of hydrodynamics at Russian universities.

In the sense of the modern kinetic theory of fluids, the book emphasizes, on the one hand, the connection and analogies between the dynamics of a viscous liquid and, on the other, the development of the theory of elastic deformations in solids.

Considered are problems tractable by means of mechanical conceptions only; questions concerning the compressibility and heat conductivity of liquids cannot be treated in books of this kind. Problems illustrating theoretical considerations are carefully worked, beginning with their mathematicophysical formulation and up to a detailed analysis of the final solution. Special attention is paid to explaining various methods for simplifying the fundamental equations of motion in order to make them suitable for approximate solution.

The book opens with an extensive introduction concerning the historical development of hydrodynamics. The actual text is divided into 12 chapters with 97 subsections, and the volume ends with a rather complete nominal index, followed by a detailed list of matters treated. There are 109 instructive figures throughout the text. The study presupposes a reasonable knowledge in solving partial differential problems by classical methods as well as by means of the Laplace transformation.

The following headings of separate chapters will give a more exact idea of the content: I. Deformation velocities of a particle. Components of stress. II. Differential equations of motion of a viscous liquid. III. General properties of motion of viscous fluids. IV. Cases of integrating exactly the differential equations of steady motion of viscous liquids. V. Motion of a viscous fluid for small Reynolds numbers. The Stokes method. VI. Hydrodynamic theory of lubrications. VII. Motion of a viscous liquid for small Reynolds numbers. The Oseen method. VIII. Theory of boundary layers. IX. Unsteady motion of a viscous incompressible fluid. X. Development of the laminar motion of liquids. XI. Stability of laminar flows. XII. Turbulent motion.

Presentation is clear and vivid, paper good, and print legible. Only a lot of unnecessary misprints and typographical oversights considerably disturb the intellectual enjoyment of reading this scientific work. Reviewer warmly recommends the book not only to specialists in the field of hydrodynamics but also to applied mathematicians and physicists.

V. Vodička, Czechoslovakia

1500. **Clarion, C., Oscillatory movement with viscosity and inertia** (in French), *Publ. sci. tech. Min. Air France* no. 303, 61 pp., 1955.

After a review on some previous work about oscillations of viscous liquids done by Grace, Sexl, Christopherson, Valembois, Menneret, and other authors, an introduction and brief description of apparatus and liquids used are presented. First part of paper deals with free oscillatory movement of a heavy viscous liquid in a U-tube. Experimental determination of period, damping coefficient, and of various regimes is reported. To visualize instantaneous shape of surface formed by particles originally at a given cross section, author applied technique of *separation surfaces*. These are obtained by pouring in one of the branches of tube, after partially filling with one liquid, another of slightly smaller density, nonmiscible with the first one, with a viscosity coefficient as close as possible and a negligible surface tension. These characteristics are very seldom available, so there is always the possibility of error in determinations. If proper care is applied, a neat surface of separation is obtained which can be photographed at every position. Author claims that this surface represents the elongations of particles in an otherwise homogeneous column. Photographs showed distinctively two regimes. First is laminar for values of $R^2\omega_p/\nu$ less than 20 (R = radius of pipe; ω_p pulsation for inviscid fluid; ν kinetic viscosity). Second regime is turbulent with laminar boundary layer for $R^2\omega_p/\nu > 70$. Between both values, an intermediate regime is observed.

Comparison between experimental results and integration of Navier-Stokes equations showed good accord for first and second regimes, as far as damping coefficient is concerned.

Second part is devoted to studies of period and damping coefficient for movement in a surge tank, with previous report on early work done by Escande, Eydoux, and Calame and Gaden. Again the separation surface technique was applied for experimental determination in a glass model. Comparison of results with theoretical integration of Navier-Stokes equations showed a remarkable agreement for pulsations and damping coefficient, for $70 < R^2\omega_p/\nu < 1000$.

Third part presents results of studies on flow of a viscous liquid through a cylindrical pipe in permanent regime, with the same photographic technique.

Some excellent photographs are reproduced.

A. Balloffet, USA

1501. **Stuart, J. T., On the role of Reynolds stresses in stability theory,** *J. aero. Sci.* **23,** 1, 86–88, Jan. 1956.

Author claims the importance of Reynolds stresses in stability theory of finite disturbance. According to his definition, a time-independent solution of laminar flow which may change to a time-dependent solution under certain conditons may still be considered as stable. This seems quite different from the ordinary concept of stability of a laminar flow.

S. I. Pai, USA

1502. **Collins, W. D., On the steady rotation of a sphere in a viscous fluid,** *Mathematika* **2,** part I, 3, 42–47, June 1955.

Author defines the function $\chi = \omega V$ for the motion due to a solid of revolution rotating slowly and steadily about its axis of symmetry in a viscous liquid at rest at infinity, where V is the

liquid velocity at a point distant ω from the axis. Since χ can be made to satisfy the same conditions as Stokes' stream function for the irrotational motion due to the same solid of revolution moving along its axis with constant speed in a perfect liquid at rest at infinity, there is an obvious analogy between the two types of problem. The steady rotation of a sphere in a viscous liquid is discussed by expanding the variables in the equations of motion as power series in R, the Reynolds number, thus obtaining the couple g on the sphere also as a power series in R. Calculations indicate that the coefficients in g decrease rapidly, so that for small values of R, the first terms should yield a good approximation. G. Power, England

1503. Von Baranoff, A., Resistance of a rotational body in accelerated or decelerated movement (in French), *Rech. aéro.* no. 45, 11–14, 1955.

This paper presents the derivation and details of the author's previously published extension [AMR **8**, Rev. 3822] of the reviewer's results [AMR **8**, Rev. 1427] for transonic drag on an accelerated body of revolution. Author's result, that the supersonic drag of a closed body could exceed its final (supersonic steady flow) value, was criticized by the reviewer [AMR **8**, Rev. 3822] as being in numerical disagreement with the results of Wong [M.S. Thesis, Univ. of Calif., Los Angeles, 1955]; moreover, the reviewer had inferred that, in consequence of *his* symmetry theorem [loc. cit.], author's results would imply negative drag for a decelerated body in some range of subsonic speeds and had regarded this as impossible. A numerical error recently has been found in Wong's results, however, while the author has informed the reviewer that his results have been subjected to an independent check. It appears, therefore, that the reviewer's earlier criticism was not justified and that the subsonic drag of a decelerated body would be negative.
Courtesy of Mathematical Reviews J. W. Miles, USA

1504. Allen, D. N. De G., and Southwell, R. V., Relaxation methods applied to determine the motion, in two dimensions, of a viscous fluid past a fixed cylinder, *Quart. J. Mech. appl. Math.* **8,** part 2, 129–145, June 1955.

The vorticity equation is transformed so that the independent variables are the velocity potential and stream function of the corresponding irrotational inviscid flow, and the resulting equation is written in finite difference form to enable an approximate numerical solution to be obtained. The distributions of the stream function and the vorticity are calculated in this way for R (Reynolds number) equal to 0, 1, 10, 100, and 1000. The distributions show several features of interest, among which is the trend, as R increases, toward a boundary-layer flow with a closed wake containing two standing eddies. No comment is made on the apparent anomaly that the authors are able to find an approximate numerical solution for $R = 0$, whereas it is believed that the analytical solution of Stokes' equation cannot be made to satisfy the boundary conditions at infinity.
G. K. Batchelor, England

1505. Chen, C.-P., Establishment of laminar flow in a channel with infinite rectangular cross section and horizontal base (in French), *C. R. Acad. Sci. Paris* **240,** 1, 48–50, Jan. 1955.

Theoretical study of the establishment of laminar flow in a rectangular channel of infinite breadth. Using boundary-layer theory and a plausible hypothesis of parabolic distribution of the velocity in the layer, a relation, in graphical form, is obtained between x_1/y_0R and F_0, with x_1 extent of establishment region, y_0 initial depth, where the velocity is uniformly distributed, R Reynolds number ($R = 4q/\nu$, q discharge for unit breadth), and F_0 initial Froude number. D. Citrini, Italy

1506. Eskinazi, S., and Yeh, H., An investigation on fully developed turbulent flows in a curved channel, *J. aero. Sci.* **23,** 1, 23–34, 75, Jan. 1956.

Measurements were made of the mean and turbulent properties in the fully developed plane incompressible flows in (a) a curved channel between circular, concentric walls and (b) a comparison straight channel; the Re for both was 74,200, based on half width. The curvature causes pronounced asymmetries in the profiles of the various properties with the higher values generally toward the outside (larger radius). Thus the pressure (of course), mean velocity, shear-stress magnitude, $\overline{u^2}$, $\overline{v^2}$ and their spectra, rate of turbulence production, and the several scales of turbulence are all relatively larger for the outer portion of the channel. The effects are discussed on the basis of the turbulent energy equations. The alteration, due to the curvature, of the energy-production terms seems to dominate the picture, with the centripetal pressure gradient playing an important role.
H. S. Ribner, Canada

1507. Chapman, A. J., and Korst, H. H., Free jet boundary with consideration of initial boundary layer, *Proc. second U. S. nat. Congr. appl. Mech.*, June 1954; Amer. Soc. mech. Engrs., 1955, 723–731.

The two-dimensional boundary-layer equations for free jet mixing with an initial boundary layer are reduced to the heat-conduction equation following the small perturbation method of S. Pai [AMR **3**, Rev. 2707]. The equations are solved in an intrinsic coordinate system with the same initial and boundary conditions as for a reference system based on a corresponding inviscid jet. The error-function velocity profiles thus obtained are solved by the momentum-integral method. Solutions are presented for various initial upstream boundary-layer forms. For turbulent mixing, Pai's method requires the experimental determination of two empirical quantities for the kinematic viscosity representation. Low-speed experimental data are reported to determine these quantities; one is apparently constant, the other a function of the initial boundary-layer parameter.
From authors' summary by F. Landis, USA

1508. Harmon, D. B., Jr., An equation for predicting a mean drop size in a high-speed spray, Univ. Calif. Publ. in Engng. **5,** 5, 145–158, 1955.

An equation is developed which can be used under specified conditions to predict the volume-to-surface mean drop size in a high-speed spray issuing from a cylindrical nozzle. The equation is obtained from an energy balance by the use of dimensionless ratios and data from other investigators. The method may be applied to finding an equation for any type of nozzle. The conclusion is drawn that drag is the fundamental drop-forming mechanism for the range of data available and the type of jet investigated. The drop diameter increases with increasing gas viscosity and decreases with increasing liquid density and surface tension. Author believes that the equation derived is suitable for the prediction of the Sauter mean drop size in a high-speed spray such as issues from a cylindrical nozzle in a diesel engine, and even to velocities as low as those occurring in rocket motor injectors. The equation is not applicable to gas pressures below atmospheric, nor to swirl sprays, in which both lineal and rotatory kinetic energies occur. K. J. DeJuhasz, Germany

1509. Brun, R. J., and Vogt, D. E., Impingemnt of water droplets on NACA 65A004 airfoil at 0° angle of attack, *NACA TN* 3586, 28 pp., Nov. 1955.

The trajectories of water droplets in the air flowing past an NACA 65A004 airfoil at an angle of attack of 0° were determined.

The amount of water in droplet form impinging on the airfoil, the area of droplet impingement, and the rate of droplet impingement per unit area on the airfoil surface were calculated from the trajectories and presented to cover a large range of flight and atmospheric conditions. These impingement characteristics are compared briefly with those previously reported for the same airfoil at angles of attack of 4° and 8°.

From authors' summary

1510. **Hawthorne, W. R., Rotational flow through cascades; Part I, The components of vorticity,** *Quart. J. Mech. appl. Math.* **8,** 3, 266–279, Sept. 1955.

The generation of vorticity in an initially nonuniform flow passing through a cascade was examined theoretically. By comparing the results obtained by considering the flow of an incompressible inviscid fluid in a force field and a vortex filament passing through a cascade, it was found that the component of vorticity in the flow direction could be divided into three parts. These are: distributed secondary circulation due to distortion of the vortex filaments, trailing shed circulation due to the change about bodies immersed in the flow, and trailing filament circulation due to the relative separation of the vortex filaments on the two surfaces of the airfoil. The second and third parts form a trailing vortex sheet separating the flow over the upper surface from the flow over the lower surface. W. Squire, USA

1511. **Hawthorne, W. R., and Armstrong, W. D., Rotational flow through cascades; Part II, The circulation about the cascade,** *Quart. J. Mech. appl. Math.* **8,** 3, 281–292, Sept. 1955.

An experimental study of the flow through a cascade of impulse turbine blades was made in which the inlet and outlet velocity profiles and pressure distribution on the blades were measured. The existence of the trailing filament circulation (see preceding review) was confirmed and its magnitude was found to be in fair agreement with the theory.

The theory was refined by taking into account the effect of the secondary flow on the circulation and lift. This tended to cause stall near the walls. W. Squire, USA

Compressible Flow, Gas Dynamics

(See also Revs. 1532, 1534, 1535, 1545, 1565, 1585, 1607, 1609, 1650)

1512. **Tao, L. N., Gas dynamic behavior of real gases,** *J. aero. Sci.* **22,** 11, 763–774, 794, Nov. 1955.

The Beattie-Bridgeman equation of state and semi-empirical equations for the specific heats c_v and c_p as functions of the temperature and pressure are assumed. First, explicit relations are derived for an isentropic change of state. Relations for the one-dimensional nonviscous isentropic flow through a Laval nozzle are then developed. Finally, relations between conditions in front of and behind a normal shock wave (modifications of the Rankine-Hugoniot equations) are derived and extended to oblique shock waves. The Crocco number, defined as the ratio of local velocity to the Laval velocity, or maximum velocity attainable in the same direction, is used as independent variable in the results. Curves show results of illustrative numerical examples, including deviations from the ideal-gas results due to the simultaneous use of the "real" equation of state and specific-heat variation. M. Morduchow, USA

1513. **Hoerner, S. V., Solutions of hydrodynamic equations with linear change of velocity** (in German), *Z. Naturforsch.* **10a,** 9/10, 687–692, Sept./Oct. 1955.

One-dimensional unsteady flow of a perfect gas ($p = R\rho T$) in absence of viscosity and heat condition is considered. Solutions of the partial differential equations (continuity, energy, and momentum) with the form $u(x, t) = A(t) \cdot x + B(t)$ are obtained, where u is the velocity, x and t coordinates of location and time, respectively, and A, B arbitrary functions of the time t alone. The result contains one arbitrary function of time in addition to three arbitrary constants. According to the choice of the constants, three types of solutions are obtained.

A particular subgroup of the above solutions is obtained by assuming that $p(x, t) = P(t)\rho^\gamma$, where p is the pressure, P a function of time alone, ρ the density, and γ a constant, given by initial conditions, in general different from the ratio of specific heats. A further particular solution is the well-known sudden expansion into vacuum of an isentropic gas initially at rest.

The obtained solutions are finally compared to the solutions across shocks. It is shown how much they deviate from exact solutions of shocks (these solutions are in preparation). The conclusion is that, in most cases, the obtained solutions give a useful approximation to shocks and, for a ratio of specific heats of 7/5, an exact solution.

Reviewer finds that the derivation of the solutions is well presented, while the comparison with the shocks is not complete. In particular, it is not clear how the assumption of no viscosity and heat conduction can lead to an exact solution across a shock. The announced publication in the same journal may clarify this point. H. P. Eichenberger, USA

1514. **Fell, J., and Lesiie, D. C. M., Second-order methods in inviscid supersonic theory,** *Quart. J. Mech. appl. Math.* **8,** 3, 257–265, Sept. 1955.

A particular integral is found for general three-dimensional steady flow. It involves double integration over only the line or surface of singularities of the first-order solution. The required kernel, or unit particular integral, is derived by ingenious combination of two different techniques. Method is shown to reproduce known particular integral for axisymmetric flow. It unfortunately becomes too complex for triple products. Second-order solutions, to be published later, have thereby been calculated for important cases of inclined axisymmetric bodies and finite wings.

M. D. Van Dyke, USA

1515. **Chou, P. T., Variational and Galerkin's methods in compressible fluid flow problems,** *J. appl. Phys.* **25,** 12, p. 1551 (Letters to the Editor), Dec. 1954.

1516. **Michael, W. H., Jr., Flow studies on drooped-leading-edge delta wings at supersonic speed,** *NACA TN* 3614, 29 pp., Jan. 1956.

An experimental investigation of the flow over flat delta wings has been extended to include studies on delta wings with drooped leading edges. Vapor-screen, pressure-distribution, and ink-flow tests were made on a series of semispan delta wings with semiapex angles of 15°, 22.5°, and 31.75° and with 10% and 20% of the semispans drooped 15° in streamwise sections. The tests were made at a Mach number of 1.9.

From author's summary

1517. **Vincenti, W. G., and Fisher, N. H., Jr., Calculation of the supersonic pressure distribution on a single-curved tapered wing in regions not influenced by the root or tip,** *NACA TN* 3499, 32 pp., June 1955.

Problem solved applies to considerable portion of many wing surfaces inclined at arbitrary angle below that for bow wave detachment. General method uses shock-expansion idea by first approximating surface by inscribed polyhedron, and then passing to limit of an infinite number of planar polyhedron elements.

Result is a pair of simultaneous nonlinear ordinary differential equations which can be integrated by numerical methods in any given case. Two examples are given for biconvex triangular-planform wing of aspect ratio 4.

Small-disturbance assumptions are applied to general method to yield separately appropriate equations for linearized supersonic flow and for hypersonic flow. Linearized form is a closed expression. Hypersonic form reduces to same result as two-dimensional-type calculation. D. R. Chapman, USA

1518. Germain, P., Subsonic flow in the neighborhood of the nose of a delta wing (in French), *Rech. aéro.* no. 44, 3–8, Mar./Apr. 1955.

It is assumed that the singularity at the nose of a delta wing in subsonic (e.g., incompressible) flow is related to that at the nose of a similar wing of infinite chord. The singular part of the flow near the nose must then be homogeneous, as defined by the same author [title source, no. 7, 3–16, 1949; AMR **3,** Rev. 103]. Making use of the theory of Fredholm integral equations, author proves that the solution of the infinite-chord problem is unique up to a multiplicative constant and homogeneous of order m, $0 < m < 1$. Furthermore, m is a continuous monotonic function of the angle 2γ formed by the leading edges at the nose and decreases from 1 to 0 as γ increases from 0 to π. When $\gamma = \pi/2$, the leading edge is straight; this is the case of plane flow and is studied in detail as part of the proof. W. R. Sears, USA

1519. Fenain, M., and Vallée, D., Application of the theory of homogeneous flows to the calculation of the wave drag of pointed, sweptback wings (in French), *Rech. aéro.* no. 44, 9–21, Mar./Apr. 1955.

This is a calculation, within the scope of linearized small-perturbation theory, of the wave drag of sweptback supersonic wings with straight leading and trailing edges and pointed tips. The theory of homogeneous flows [see Germain, title source no. 7, 3–16, 1949; AMR **3,** Rev. 103] is employed. Two methods of synthesizing wings are available: superposition of homogeneous flows having the same vertex and planform, and "composition," which is defined as the superposition of flows having different vertexes and planforms. The former is carried out in detail and leads to formulas for the wave drag involving sets of functions, some of which are tabulated here and in an earlier paper [Fenain, title source, no. 16, 27–38, 1950; AMR **4,** Rev. 2117].

Application of the theory is made to a family of wings whose surface is defined by a function of third degree. Some geometrical properties of the members of this family are worked out. The family includes, for example, wings with affine profiles of third degree. A variety of cases has been worked out numerically and the results are presented in graphs. These cases have been selected to illustrate the effects of the several parameters defining the wing surfaces. W. R. Sears, USA

1520. Napolitano, L. G., and Ferri, A., Application of the linearized characteristics method to the determination of the axisymmetric supersonic flow near the nose of a body of revolution, *Poly. Inst. Brooklyn, PIBAL Rep.* no. 279, 23 pp. + 8 figs. Mar., 1955.

Ferri's method [AMR **5,** Rev. 1137] is applied to ogive differing only slightly from circular cone. Initial gradients of flow quantities at tip (neglecting vorticity) are considered in detail; extension to higher terms is outlined. Numerical results are given for ogive $r = 0.1333(x - x^2 - \ldots)$ at 3.18 Mach number, but are unfortunately misplotted. Corrected point falls nicely between reviewer's curve for vanishing thickness [AMR **7,** Rev. 3952] and curve for 10° semi-vortex angle calculated earlier by Shen and Lin [AMR **5,** Rev. 1134]. Interesting analysis determines when linearization fails because of formation of envelopes of characteristic lines. M. D. Van Dyke, USA

1521. Parker, H. M., Minimum-drag ducted and pointed bodies of revolution based on linearized supersonic theory, *NACA Rep.* 1213, 9 pp., 1955.

See AMR **7,** Rev. 3626.

1522. Ford, C. A., and Glass, I. I., On the interaction of two similarly facing plane shock waves, *J. appl. Phys.* **25,** 12, 1549–1550 (Letters to the Editor), Dec. 1954.

1523. Glass, I. I., and Martin, W. A., Experimental and theoretical aspects of shock-wave attenuation, *J. appl. Phys.* **26,** 1, 113–120, Jan. 1955.

See AMR **8,** Rev. 2433.

1524. Stanbrook, A., Lift curve slope at subsonic and supersonic speeds. Graphical presentation of theoretical results for straight-tapered wings, *Aircr. Engng.* **26,** 306, 244–246, 257, Aug. 1954.

Theoretical results for the initial lift-curve slope of straight-tapered wings, based on the linearized equations of inviscid motion, have been evaluated and are presented in the form of graphs. From author's summary

1525. Merle, M., Structure of a high-speed air jet (in French), *C. R. Acad. Sci. Paris* **240,** 20, 1972–1975, May 1955.

This note is meant to explain the differences in results published by Outart and by Hartmann and Lazarus about the length of periodical shock configurations in free jets. It points out that three pressure ratio regions exist with essentially different shapes of the shocks. S. F. Erdmann, Holland

1526. Tucker, N. B., An experimental investigation of flow separation in a nozzle at high Mach numbers and low density, *Univ. Toronto Inst. Aerophys. Rep.* **33,** 33 pp. + 4 tables, 18 figs., 12 plates, Aug. 1955.

Investigation of air flow in supersonic nozzles showed that at Reynolds numbers of 20,000 to 150,000 (based on experimental exit-flow conditions and length of supersonic portion of nozzle and obtained in a 5-in. closed jet wind tunnel at atmospheric stagnation pressure), separation of flow from nozzle walls restricted the attainable Mach number. Separation occurred in two-dimensional, sharp, and rounded throat nozzles and in smooth throat, square, axisymmetric nozzles. The onset of flow separation in a round throat, two-dimensional wedge nozzle was delayed from $M = 3.8$ to $M = 5.7$ by application of surface roughness on the nozzle flow surfaces.

The report also contains information on measurement of high Mach numbers and detection of air condensation in high-speed flow. J. Lukasiewicz, Canada

1527. Heybey, W. H., and Reed, S. G., Jr., Weak detonations and condensation shocks, *J. appl. Phys.* **26,** 8, 969–974, Aug. 1955.

Authors believe rapid condensation processes encountered in expanding gases are examples of weak detonations. Normal condensation shocks for steady supersonic airstream containing water vapor and rapid condensation of steam are discussed. Authors compare kinematics of weak detonations with shock-tube experiments.

Condensation processes are compared with Fredrich's theoretical work on chemical weak detonations. Authors conclude rapid

condensation processes or weak detonation transitions are not initiated by a shock as in chemical detonations.

F. I. Given, USA

1528. **Woods, L. C., On the theory of two-dimensional wind tunnels with porous walls,** *Proc. roy. Soc. Lond.* (*A*) **233,** 1192, 74–90, Dec. 1955.

A mathematical theory is developed for the design of wind tunnels with porous walls to give zero tunnel blockage in subsonic flow. Blockage refers to the velocity increment at the model caused by the tunnel walls. The tunnel walls are porous over a finite length and solid elsewhere. A sealed jacket is placed over the porous length to provide pressure control.

It is found that, for a given porosity, the length of the porous wall must be reduced with increasing Mach number to keep the blockage zero. Both solid and wake blockage are considered. Wake blockage cannot be eliminated by reducing the length of the porous wall alone. The rate of removal of air from the sealed jacket can be adjusted to eliminate the induced pressure gradient completely.

R. C. Binder, USA

Wave Motion in Fluids

(See also Revs. 1498, 1559)

1529. **Kazarnovskiĭ, Ju. E., Flood wave transformation in reservoirs with siphon spillways** (in Russian), *Gidrotekh. i Melior.* **7,** 8, 55–57, Aug. 1955.

Effect of reduction of flood crest is more significant in a reservoir with automatic release than with overflow spillway. A simple flood-routing procedure is explained.

S. Kolupaila, USA

1530. **Apté, A., and Marcou, C., Model study of agitation of water damped with oil** (in French), *C. R. Acad. Sci. Paris* **241,** 24, 1702–1704, Dec. 1955.

Turbulence, Boundary Layer, etc.

(See also Revs. 1506, 1507, 1543, 1606, 1658, 1659)

1531. **Spence, D. A., The development of turbulent boundary layers,** *J. aero. Sci.* **23,** 1, 3–15, Jan. 1956.

Paper presents a method for computing the development of turbulent boundary layers in pressure gradients, based on a correlation of observed mean velocity profiles. A number of published measurements have been analyzed to show that, within a momentum thickness of the wall, the velocity distribution follows the universal velocity distribution and, beyond this, a power law. Physical justification of this result is found in recent measurements of the turbulent motion. From the equation of mean motion in the inner layer and making some simplifying approximations, an equation for the rate of change of the shape parameter is obtained, somewhat similar to that proposed by von Doenhoff and Tetervin. A procedure for the numerical solution of the equations is described and the method is applied with success to layers on two airfoils for which measurements are available. Generally, the method appears relatively convenient in use and predicts plausible movements of the point of separation with increase of Reynolds number and of initial thickness.

A. A. Townsend, England

1532. **Bogdonoff, S. M., and Kepler, C. E., Separation of a supersonic turbulent boundary layer,** *J. aero. Sci.* **22,** 6, 414–424, 430, June 1955.

Authors report results of two series of experiments on supersonic turbulent boundary layers on a flat plate, using forward-facing steps and impinging shock waves, respectively, to produce adverse pressure gradients. Reported measurements include surface pressure distributions and total head profiles in the interaction region. Results show that for "weak" disturbances (small or no separation region) the phenomena depend strongly on the strength and kind of disturbance, whereas for "strong" disturbances (large separation region) the shape of initial pressure rise and pressure ratio at separation are practically the same for both types of disturbance. Increasing the strength of disturbance merely moves this initial region forward. The disturbance strengths for which interaction becomes strong in this sense are about 2 boundary thicknesses for step height and a pressure ratio of about 2.5 for impinging shock, respectively. Pressure ratio at separation is about 2. After separation, the pressure continues to rise, the extent and shape of this further rise depending on the nature and strength of disturbance.

Data from this and other investigations are cited to show small effect of Mach number and Reynolds number on the separation pressure ratio.

A. Roshko, USA

1533. **Hill, F. K., Boundary-layer measurements in hypersonic flow,** *J. aero. Sci.* **23,** 1, 35–42, Jan. 1956.

Boundary-layer measurements were made on the wall of a conical nozzle, in presence of negative pressure gradients and several rates of heat transfer, at Mach numbers 8.25 and 9, and Reynolds numbers based on momentum thickness from 1500 to 3500. An important and new result is that the laminar sublayer is relatively thick (~15% of total boundary-layer thickness) at hypersonic Mach numbers and that the physical process of heat transfer, in this case, may be different from that in boundary layers where turbulence predominates through 96 to 99% of the total thickness. Wall shearing stress is derived from the velocity profile by boundary-layer surveys of pitot pressure, static pressure, and total temperature; the results concerning skin-friction coefficients indicate agreement to within 5% with a form of the Wilson theory [title source **17,** 9, 585–594, Sept. 1950].

A. E. Brun, France

1534. **Lew, H. G., and Fanucci, J. B., On the laminar compressible boundary layer over a flat plate with suction or injection,** *J. aero. Sci.* **22,** 9, 589–597, Sept. 1955.

Authors extend the work of Iglisch [*NACA TM* 1205] to include the effects of compressibility and linear variations with temperature of viscosity and thermal conductivity. They show that these results, of a numerically integrated boundary layer, compare favorably with those of momentum methods.

Asymptotic solutions, neglecting variations with distance along the plate, are developed, and it is shown that the above solutions approach the asymptotic solutions in a short distance along the plate.

M. W. Rubesin, USA

1535. **Hakkinen, R. J., Measurements of turbulent skin friction on a flat plate at transonic speeds,** *NACA TN* 3486, 41 pp., Sept. 1955.

Present report describes the design and construction of a floating-element skin-friction balance. This instrument, which is essentially an improved version of Dhawan's balance, was applied to measurements of local skin friction in the turbulent boundary layer of a smooth flat plate at high subsonic Mach numbers and supersonic Mach numbers up to 1.75. The measured skin-friction coefficients are consistent with the results of other investigations at subsonic and also at supersonic speeds. The principal difficulties which exist in comparing skin-friction coefficients at various Mach numbers are discussed.

From author's summary by Y. K. Jan, Holland

1536. Shapiro, A. H. Siegel, R., and Kline, S. J., Friction factor in the laminar entry region of a smooth tube, *Proc. second U. S. nat. Congr. appl. Mech.*, June 1954; Amer. Soc. mech. Engrs., 1955, 733–741.

Paper treats theoretical and experimental investigations of the development of the laminar boundary layer and the transition to Poiseuille flow in the inlet region of a bell-mouthed cylindrical tube of circular cross section. Experiments were carefully performed, and they provide valuable data on the local apparent friction factor in the inlet region of a tube.

Two different approximate analyses are presented. The first applies for the region near the inlet, in which the boundary-layer thickness is small compared to the radius of the tube. It employs an iteration process based on Thwaite's approximate method for calculating the laminar boundary layer. The second analysis utilizes integral forms of the momentum equation and the continuity equation, in conjunction with cubic or quartic polynomial approximations of the velocity profile. It provides results that are applicable throughout the entire transition length.

Theoretical results are compared with experimental data and with theories of other investigators.

H. L. Langhaar, USA

1537. Lachmann, G. V., Practical application of boundary layer control (in German), *Z. Flugwiss.* **4,** 1/2, 9–14, Jan./Feb. 1956.

Methods which are already in use or are being developed for the purpose of preventing separation of the boundary layer on airfoils and the increase of lift due to flaps by sucking or blowing are critically reviewed. Some fundamental questions concerning the practical application of suction to stabilize a laminar boundary layer are discussed, and the influence of reduction of drag and weight on the direct operating costs of long-range airliners is demonstrated. From author's summary

1538. Stephens, A. V., and Collins, G. A., Turbulent boundary layer control by ramps or wedges, *Austral. aero. Res. Comm. Rep.* no. ACA-55, 19 pp., May 1955.

See AMR **8,** Rev. 1091.

1539. Mickelsen, W. R., An experimental comparison of the Lagrangian and Eulerian correlation coefficients in homogeneous isotropic turbulence, *NACA TN* 3570, 42 pp., Oct. 1955.

Values of the turbulent spreading coefficient ω_T were determined by injecting helium into the central core of a turbulent pipe flow and measuring concentration profiles. By fitting each profile with an error function, the value of ω_T at different downstream stations (that is, different values of t), was determined from Taylor's diffusion theory, which defines $\omega_T(t)$ as the double integral, up to t, of the Lagrangian correlation coefficient R. The latter is not conveniently measurable.

The Eulerian correlation coefficient f was measured by standard hot-wire technique. Its double integral, up to ξ, is called $\omega_f(\xi)$ in analogy to ω_T. The measurements show that $\omega_T(t)$ and $\omega_f(\xi)$ have similar shapes, implying that the correlation coefficients $R(t)$ and $f(\xi)$ have similar shapes, that is, $f(\xi) = R[\xi/B(\bar{v}^2)^{1/2}]$, where $(\bar{v}^2)^{1/2}$ is the rms value of the transverse turbulent velocity component. From the experiments, B is roughly 0.6. Thus a measurement of f makes it possible to infer R, which is the coefficient of interest in many practical mixing problems.

A. Roshko, USA

1540. Hutchings, J. W., Turbulence theory applied to large-scale atmospheric phenomena, *J. Meteor.* **12,** 3, 263–271, June 1955.

Data taken at the earth's surface, 18,000 ft, and 30,000 ft are given. The theoretical results, giving the form of the correlation function for velocity fluctuation according to the thory of Kolmogoroff, are applicable to eddy sizes large enough to affect weather observations at a point. J. C. Freeman, USA

1541. Saissac, J., Homogeneous turbulent diffusion of particles (in French), *J. sci. Météor.* **7,** 26, 227–232, Apr.-June 1955.

Author analyzes data on atmospheric diffusion of a smoke cloud in terms of the classic diffusion equation with time-dependent diffusivity. The solution has, of course, the same analytical appearance as that for constant diffusivity. The central variable is an apparent radius, presumably related to the projected cloud opacity as seen at a fixed observation point. Anomalous experimental results such as timewise decrease in apparent diffusivity are attributed to the observation method, to the decoy of an explosion-induced velocity disturbance, and to the differing scales of atmospheric turbulence.

The experimental procedures and the apparent radius concept are not described in enough detail to permit evaluation by the reviewer. S. Corrsin, USA

1542. Uberoi, M. S., and Kovasznay, L. S. G., Analysis of turbulent density fluctuations by the shadow method, *J. appl. Phys.* **26,** 1, 19–24, Jan. 1955.

From very accurate two-dimensional space correlations of the transparency of spark shadow photographs, the three-dimensional space correlations of turbulent density fluctuations in the wake of a supersonic projectile and a "Swiss cheese" grid projectile, and in a low-speed heated jet were calculated (assuming axial symmetry and homogeneous, isotropic turbulence). In the latter case, comparison with hot-wire anemometer measurements was made, showing good agreement in the microscale and fair agreement in the integral scale. Method appears useful in finding small-scale eddy structure at high speeds, which is difficult for the hot wire. A. E. Bryson, USA

Aerodynamics of Flight; Wind Forces

(See also Revs. 1351, 1357, 1358, 1509, 1516, 1517, 1521, 1524, 1537, 1560, 1562, 1563, 1564, 1582, 1584, 1585, 1646)

1543. Wattson, R. K., Jr., Effects of boundary-layer control system efficiency on performance, *Aero. Engng. Rev.* **14,** 12, 49–51, Dec. 1955.

An approach to the evaluation of the effect of boundary-layer control system efficiency on climb, take-off acceleration, and range is presented, utilizing the energy concept. The results are not directly applicable to a particular aircraft without prior knowledge of the individual characteristic boundary-layer control effects on lift and drag, as is pointed out. They do, however, enable visualization of the effect of the efficiency of the pumping system, i.e., its capacity to convert fuel energy into work on the boundary-layer air, or system input horsepower into air horsepower.

From author's summary by R. M. Stewart, USA

1544. Laporte, O., and Yoshihara, H., A rigorous method for finding the lift of certain class of airfoils and remarks on the meaning of Schrenk's approximate rule, *J. aero. Sci.* **22,** 11, 787–794, Nov. 1955.

Paper gives exact solutions to the lifting-line theory for a family of straight wings with different aspect ratios and tapers including symmetrical twists. The method used is based on a conformal mapping onto a different plane of the Trefftz wing plane. In the new plane the adequate selection of the mapping function allows simplification of the boundary conditions of the

Trefftz problem. Thus, the reaching of exact solutions is possible. The mapping function determines the family of wings analyzed. The method is based on a work previously published by the senior author [*Quart. appl. Math.*, Oct. 1944].

A plausible approximation to the exact solutions obtained leads to the well-known Schrenk empirical formula for straight wings without twist and provides rational basis for the same. Simple interpretation of this approximation allows it to be generalized to wings with arbitrary planforms and twists.

Reviewer believes that this work has both practical and theoretical interest since wings analyzed cover a broad family of practical forms; and it increases considerably the set of wings for which exact solutions are available, previously limited to elliptic chord distributions. G. Millán, Spain

1545. **Polhamus, E. C., Summary of results obtained by transonic-bump method on effects of plan form and thickness on lift and drag characteristics of wings at transonic speeds,** *NACA TN* 3469, 33 pp., Nov. 1955.

Paper presents a summary of the effects of planform and thickness on the lift and drag characteristics of wings at transonic speeds and comparisons with subsonic, transonic, and supersonic theories. The data considered in this summary were obtained during a transonic research program conducted in the Langley high-speed 7 by 10-ft tunnel by the transonic-bump method. The Reynolds numbers of the tests were generally less than 1×10^6. From author's summary

1546. **Kafka, P. G., Lifting pressure on delta wings with subsonic leading edges, symmetrical plan form, and discontinuous slope,** *J. aero. Sci.* **22,** 10, 725–726 (Reader's Forum), Oct. 1955.

The Lomax-Heaslet theory [*NACA TN* 2497, Sept. 1951; AMR **6,** Rev. 821] for finding the lifting pressure on delta wings is extended to include cases when $p \neq 0$ and q is an odd number. T. Y. Toong, USA

1547. **Jacobs, W., Downwash behind wings at supersonic speeds. A simplified method for calculation and experimental results for wings with small aspect ratio,** *Aero. res. Inst. Sweden Rep.* no. 61, 52 pp., 1955.

Paper presents a linearized theory for the downwash at supersonic speeds in the wake of an unswept wing of high aspect ratio. Essential for the method is the replacement of the wing by a single bound vortex. Then the entire problem is defined by the vortex distribution in the wake. This quantity is considered as known; it can be found from the pressure distribution over the wing or by measurement. The following numerical procedure is adopted: First the downwash for some standard vorticity distributions is expressed in analytical form and by means of graphs. Next the vorticity distribution belonging to a given wing, and subsequently also its downwash, is approximated by a superposition of these standard distributions. Comparison with other theoretical approaches and with experiments gives a good approximation even though the basic assumptions of the theory are not fulfilled too well. G. Guderley, USA

1548. **Pringle, G. E., and Somerville, T. V., Wing parachutes for recovery from the spin, Part I. General design requirements; Harper, D. J., Mitchell, J. R., Picken, J., and Pringle, G. E., Part II, Wake phenomena,** *Aero. Res. Counc. Lond. Rep. Mem.* no. 2543, 10 pp., 1954.

1549. **Decker, J. L., Criteria for longitudinal stability requirements,** *J. aero. Sci.* **22,** 9, 639–644, Sept. 1955.

1550. **Kraft, C. C., Jr., Theoretical analysis of an airplane acceleration restrictor controlled by normal acceleration, pitching acceleration, and pitching velocity,** *NACA TN* 3243, 42 pp., Sept. 1954.

Normal acceleration of an airplane is limited by means of a mechanical brake on the elevator control system. This brake is actuated if a signal proportional to normal and pitching acceleration exceeds a preset value. Rate of elevator motion is 30°/sec. Results were obtained by use of an electronic analog computer. Ratio of peak to preset acceleration with no time lag in the mechanism was about 1.4 and remained constant with increasing airspeed. Addition of pitching velocity gave still better performance, but increasing the time lag to 0.05 sec made performance unsatisfactory. W. Oppelt, Germany

1551. **Hooke, F. H., The influence of aeroplane characteristics on the response to gusts of various forms,** *Aero. Res. consult. Comm. aero. Res. Lab. Melbourne, Austral. Rep.* 54, 1–35, Aug. 1954.

1552. **Cheatham, D. C., A study of the characteristics of human-pilot control response to simulated aircraft lateral motions,** *NACA Rep.* 1197, 14 pp., 1954.

1553. **Pratt, K. G., and Walker, W. G., A revised gust-load formula and a re-evaluation of V-G data taken on civil transport airplanes from 1933 to 1950,** *NACA Rep.* 1206, 9 pp., 1954.

See AMR **7,** Revs. 245, 1904.

1554. **Laitone, E. V., Limiting velocity by momentum relations for hydrofoils near the surface and airfoils in near sonic flow,** *Proc. second U. S. nat. Congr. appl. Mech.*, June 1954; Amer. Soc. mech. Engrs., 1955, 751–753.

It is shown that the surface velocity over the upper surface of a two-dimensional hydrofoil can have a definite limiting magnitude as the hydrofoil approaches the free water surface of infinitely deep water. This limiting condition is governed by the maximum depth available from the hydraulic jump relations.

Then it is shown that an analogous limiting surface velocity can exist on a two-dimensional airfoil at high subsonic speeds whenever a local supersonic flow is terminated by a normal shock wave. This limit in the magnitude of the surface velocity occurs at the termination of the local supersonic flow region and is similarly governed by the normal shock wave relations. From author's summary by M. J. Goglia, USA

Aeroelasticity (Flutter, Divergence, etc.)

1555. **Lane, F., System mode shapes in the flutter of compressor blade rows,** *J. aero. Sci.* **23,** 1, 54–66, Jan. 1956.

Author gives a general method for investigating flutter of a multi-bladed system where each blade has two or more degrees of freedom. He shows that a system of identical blades identically supported and equally spaced on a common rotor can flutter with identical modes in which each blade leads or lags its neighbor by a common phase angle, the value of the angle depending on the configuration considered. Thus the problem is reduced to one of investigating the flutter of a "single equivalent blade." Analysis is extended to the case where the rotor oscillates in the rotary direction. Author shows that under certain conditions this mode of oscillation can be neglected without introducing errors.

Attention is drawn to earlier work on flutter of cascades of airfoils and to the analogous problem of wind-tunnel interference effects on oscillating airfoils. Author intends, in a further report,

to consider interference effects on a blade system in which the blades oscillate with constant phase shift arbitrary value between adjacent blades. W. P. Jones, England

1556. Rowe, J. R., and Mendelson, A., Experimental investigation of blade flutter in an annular cascade, *NACA TN* 3581, 24 pp., Nov. 1955.

The stall operating region of an annular cascade was investigated under varied conditions of geometry and air flow. Controlled variables were angle of attack, blade chord and spacing, number of blades in the cascade, and air velocity. Geometric angle of attack was varied between 0° and 65° with air flows to 500 fps. Several blades in each cascade were equipped with strain gages to determine stress, phasing, and flutter frequency. Pressure probes and hot-wire anemometers were used to trace air flows and fluctuations in the test section.

Stall flutter was the only flow phenomenon observed. Rotating stall, which often occurs during stalled operation of many compressors, was not observed, although attempts were made to obtain it. Test results indicate that vibratory stress peaks sharply at an angle of attack immediately above the peak mean stress (constant bending stress upon which the vibratory stress is superimposed). This is also approximately the angle of minimum aerodynamic damping. Thus, the decreased damping at stalling angle of attack allows the blades to vibrate sporadically with high stresses under the minor random flow fluctuations of the air stream. Some serious vibratory stresses are shown to exist even prior to blading stall.

Many blades had to be replaced in the cascade as tests continued and blades fatigued to fracture. It appears that the peak stresses are increased with increased chord. Decreased blade spacing tended to raise the angle of attack at which stress peaks occur. There was no constant phasing between any blades; the flutter amplitudes were sporadic and unpredictable with respect to time. The blades fluttered in the fundamental bending mode practically to the exclusion of other modes.

From authors' summary by J. R. Schnittger, Sweden

1557. Waller, M. D., Air circulations about a vibrating plate, *Brit. J. appl. Phys.* **6,** 10, 347–348, Oct. 1955.

Fine powders are employed, as in Faraday's experiments, to observe the air circulations above a vibrating plate. The surface streaming, which occurs away from nodal lines and also inward from the edges, is recorded by means of a mixed heterogeneous powder. Surface streamings from the side edges of a bar give rise to four circulations; compare results of other experiments in which there is relative oscillatory motion between a solid, e.g., a cylinder or reed, and a fluid. The senses of these circulations are, in the present instance, the same as those of Andrade's fourfold vortexes. From author's summary

1558. Fung, Y. C., The flutter of a buckled plate in a supersonic flow, *Cal. Inst. Technol. Guggenheim Aero. Lab.* OSR-TN-55-237, 23 pp. + 9 figs., July 1955.

Previous analyses of Hayes [*No. Amer. Aviation Rep.* AL-1029, May 10, 1950] and reviewer [ibid., *Rep.* AL-1140, Oct. 18, 1950] assumed amplitude of buckle large compared with panel thickness and neglected change in length of plate. As in his earlier analysis of static problem [AMR **8,** Rev. 1333], author drops these restrictions and, retaining reviewer's low-frequency approximation to aerodynamic forces (thereby ruling out *linear* analysis in $M > 2^{1/2}$), studies dynamic stability of: (a) perturbation about static (*with* aerodynamic forces) configuration of flat plate; (b) perturbation about buckled configuration; (c) nonlinear limit cycle of van der Pol type. Results of (a) reveal dynamic stability is possible if dynamic pressure between certain finite limits and size of buckle *in absence of aerodynamic forces* is sufficiently small (*this* case excluded from Hayes/Miles analyses in consequence of their initial assumptions). Results of (b) agree with those of reviewer [loc. cit. ante] in predicting dynamic instability only for dynamic pressure in excess of critical value prescribed by requirement for static equilibrium and $M > 2^{1/2}$ (the latter being a rather misleading consequence of low-frequency aerodynamic approximation). Results of (c), obtained by Kryloff-Bogoliuboff method, provide indirect solution for limit cycle in which amplitude limited by nonlinear increase in stiffness is due to stretching of plate; this contrasts with reviewer's solution [loc. cit. ante] for limit cycle, where amplitude is limited by constraint of constant length. Author's nonlinear solution appears more realistic than reviewer's, but further analysis and experiment appear desirable. J. W. Miles, USA

1559. Allen, W. A., Mapes, J. M., and Mayfield, E. B., Shock waves in air produced by elastic and plastic waves in a plate, *J. appl. Phys.* **26,** 1, 125–126 (Letters to the Editor), Jan. 1955.

1560. Fisher, L. R., Lichtenstein, J. H., and Williams, K. D., A preliminary investigation of the effects of frequency and amplitude on the rolling derivatives of an unswept-wing model oscillating in roll, *NACA TN* 3554, 29 pp., Jan. 1956.

A model with separable wing and tail assembly was oscillated in roll through a range of frequencies and amplitudes of oscillation for an angle of attack of 0° and at one frequency and amplitude for two higher angles of attack in order to determine the effects of the unsteady motion on the rolling stability derivatives of the model and its components.

A variation of frequency or amplitude of oscillation in the range covered at an angle of attack of 0° had no important effect on either the yawing moment due to rolling or the damping in roll for this unswept-wing airplane configuration. The only appreciable value of yawing moment due to rolling was shown by the fuselage-tail combination. This configuration experienced a reduction in magnitude of the derivative as either the frequency or the amplitude of the oscillation increased.

The values of the rolling derivatives obtained by oscillation were consistent with the values measured by means of conventional rolling-flow tests at an angle of attack of 0°. For the model with the wing at a high angle of attack, the oscillatory yawing moment due to rolling was different from that obtained under steady-state conditions. From authors' summary

1561. Pines, S., Dugundji, J., and Neuringer, J., Aerodynamic flutter derivatives for a flexible wing with supersonic and subsonic edges, *J. aero. Sci.* **22,** 10, 693–700, Oct. 1955.

A box method is developed for obtaining the generalized air forces on an oscillating flexible wing in supersonic flow with both supersonic and subsonic edges. Essentially, the method consists of representing the wing by a grid of square boxes and determining the influence of one box on another. These aerodynamic pressure influence coefficients, when tabulated, permit the flutter analysis of an arbitrary wing with arbitrary normal modes to be carried out in a routine way. The coefficients satisfy the linearized unsteady supersonic-flow equations and the downwash boundary conditions, and, in addition, are formulated in a manner independent of the modal shapes of the structure.

The box method is applied to some simplified examples involving rigid body modes, and agreement with other methods is seen to be reasonably good.

The box procedure appears to offer a simple routine manner of

analyzing flexible wings for supersonic flutter analyses which is well adapted to programming on computing machinery. However, the square box method as developed here for subsonic edges is inapplicable to Mach numbers below $M = 1.414$ without further modification. Some suggested modifications for extending below the $M = 1.414$ range are discussed.

From authors' summary by S. J. Zand, USA

1562. **Mazet, R., Representation of free vibration of wings in a wind-stream using the notion of response to unit impulse** (in French), *ONERA TN* 19, 25 pp., 1954.

Author presents a semiemprical method for evaluating the nonsteady aerodynamic forces on an oscillating airfoil. The method is applied to both the two-dimensional case and to the finite span wing of arbitrary planform. Application to flutter is illustrated.

J. Lorell, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 1359, 1360, 1371, 1469, 1493, 1496, 1497, 1510, 1511, 1555, 1579, 1593, 1594, 1596, 1597, 1653)

1563. **Payne, P. R., The stiff-hinged helicopter rotor,** *Aircraft Engng.* **27,** 321, 358–370, Nov. 1955.

Author suggests and investigates a rotor system with elastic restraints in the flapping hinge. Such a system shows all the aerodynamic advantages of a conventional rotor with high offset of the flapping hinges but avoids its mechanical complexity. In particular, positive stability in hovering can be achieved, and there is no limitation to maximum forward speed other than shock stall and bending strength of the blades.

The use of drag hinge stiffness is also briefly discussed. Above a certain stiffness no mechanical damping is needed to eliminate ground resonance. H. Parkus, Austria

1564. **Falabella, G., Jr., and Meyer, J. R., Jr., Determination of inflow distributions from experimental aerodynamic loading and blade-motion data on a model helicopter rotor in hovering and forward flight,** *NACA TN* 3492, 184 pp., Nov. 1955.

"Equivalent" inflow velocities were determined from pressure distribution and blade-motion measurements on a 5-ft diam rotor by means of standard blade-element rotor theory. Data were obtained at tip-speed ratios ranging from 0.10 to 1.0 with untwisted blades having zero and 13% flapping-hinge offset. The resulting plots of equivalent inflow velocity as a function of azimuth angle and blade span are useful in checking qualitatively the rotor inflow theories of Coleman, Mangler, and Castles and in comparing the inflow patterns obtained with zero and 13%-offset rotors under the same conditions of operation. Since no cyclic pitch was applied to relieve the hub moments resulting from flapping-hinge offset, the data indicate the effect of hinge offset in delaying retreating-blade stall. In interpreting the data, it should be borne in mind that the inflow velocities were calculated on the basis of a constant two-dimensional lift curve slope, an assumption which would yield questionable results in the region of the blade tips and in those parts of the disk undergoing stall.

A. Gessow, USA

1565. **Nobile, U., A graphical method for the solution of the equation of motion in the simple momentum propeller theory in a perfect compressible fluid** (in Italian), *Aerotecnica* **35,** 4, 164–170, Aug. 1955.

Except for some differences in defining the value of the power input, writer derives fundamental equations equivalent to those given by L. Talbot and A. K. Oppenheim [*J. aero. Sci.* **21,** 1, p. 61, 1954; AMR **7,** Rev. 2535], whose note is not cited in the list of references. Talbot and Oppenheim proposed some kind of iteration process of solving system of equations, whereas Nobile applies graphical method. Included are diagrams of power, density coefficients, influence of Mach number, etc. There are misprints in Eqs. (5) and (6) (should be $V_1 - V_2$).

M. Z. Krzywoblocki, USA

1566. **Gray, W. H., Hallissy, J. M., Jr., and Heath, A. R., Jr., A wind tunnel investigation of the effects of thrust-axis inclination on propeller first-order vibration,** *NACA Rep.* 1205, 37 pp., 1954.

Data on the aerodynamic excitation of first-order vibration occurring in a three-blade propeller having its thrust axis inclined to the air stream at angles of 0°, 4.55°, and 9.80° are presented. For several representative conditions it was found that the aerodynamic exciting force that results from the propeller inclination may be calculated with adequate accuracy at low rotational speeds, but with decreasing accuracy as section velocities approach $M = 1$. At low rotational speeds, the first-order vibratory stress was proportional to the product of thrust-axis inclination and dynamic pressure. At high rotational speeds, the stresses were lower than those estimated by a linear extrapolation of the low-speed stresses. A semiarticulated propeller indicated stresses considerably reduced from those for a rigid propeller.

A. Gessow, USA

1567. **Morgan, W. B., Open water test series of a controllable pitch propeller with varying number of blades,** *David W. Taylor Mod. Basin Rep.* no. 932, 5 pp. + 7 figs., Nov. 1954.

A series of open-water propeller tests was conducted with an adjustable pitch propeller with 2, 3, 4, 5, and 6 blades. The faired coefficient curves of thrust, torque, and efficiency are presented with the propeller design. An example is given to illustrate the use of the curves in selecting the optimum propeller for a given set of conditions. From author's summary

1568. **Ross, C. C., and Banerian, G., Some aspects of high suction specific speed pump inducers,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-124, 11 pp. + 11 figs.

This valuable paper, an elaboration of the article of the senior author, "Principles of rocket-turbopump design" [*J. Amer. Rocket Soc.*, Mar. 1951], describes within security restrictions the later developments of the cavitating inducers for much higher suction specific speeds for rocket application. The purpose of the rather inefficient inducer is to provide just enough head for the main impeller to make it noncavitating and thereby efficient. Authors develop some useful design parameters and graphically represent them as functions of suction specific speed, but point out that optimization of design, also effect of major scale differences and particularly the best method of limiting cavitation damage, await further research. Reviewer would make the same statement about the whole subject regarding industrial applications, where long life is more important than weight and size.

A. Hollander, USA

1569. **Bowerman, R. D., Effect of the volute on performance of a centrifugal pump impeller,** Hydrodyn. Lab. Calif. Inst. Techn., Pasadena, Calif., Rep. no. E-19.7, 21 pp., Mar. 1955.

Portion of basic centrifugal pump research program. Three two-dimensional twin volume shapes are successively applied and the effects compared with previous work on same impeller in vaneless diffuser. Volute shapes are formed by thin angle-curved sheet vanes following logarithmic spiral curves in flat shrouds. Author

finds little effect from vanes compared to vaneless diffuser operation at flows of same spiral angles. Far from design flow volute vanes induce changes in impeller performance, producing large changes in pump performance. Flow entering volute is nonuniform and resulting velocity distributions are markedly different from potential flow. In spite of above, volute passage losses are found to be only 3% of pump head at design point. Flow based on velocity measurements at volute discharge was 5 to 8% higher than measured flow.

Reviewer considers author's volute form is not representative of general practice, which takes advantage of volute to diffuse in less radial distance than possible with vaneless diffuser. Nonuniformities found in volute are suspected by reviewer to exist undetected in vaneless diffuser tests. Reviewer deems author's total head probe is subject to falsely high reading in pulsating flow conditions. More work is needed before results are directly applicable to commercial design. F. C. Gilman, USA

1570. Wright, A. W., and Szomanski, E., Application of the Lighthill method to the test cascade design, *Aero. Res. Lab. Melbourne, Austral. Mech. Engng. Note* no. 201, 40 pp. + 7 figs., June 1955.

An attempt has been made to expand the Lighthill method of cascade design in such a manner that the newcomer to the field, providing he has a grounding in conformal transformations and complex variable theory, may proceed to its successful application to design, substantially without tutorial assistance.

From authors' summary by S. I. Pai, USA

1571. Crewdson, E., Water-ring self-priming pumps, *Inst. mech. Engrs.* Preprint, 13 pp., 1955.

The method of operation of water-ring self-priming pumps is briefly described with performance data of three designs of such a pump. The mechanism by which these pumps produce their effect is discussed, and the conclusions reached indicate that it is impossible to anticipate their performance from their physical dimensions as can be done with a centrifugal pump. If, however, the head-discharge curve of any pump is established, and the horsepower required to overcome mechanical and disk friction is known or can be closely estimated, the horsepower taken by the pump at any point of its range can be calculated with a high degree of accuracy. Included in the information are graphs of air-pumping capacity and details of experiments to determine the best designs of rotor vane. Small changes in the shape of the vanes produce remarkable differences in performance.

From author's summary

Flow and Flight Test Techniques

(See also Revs. 1523, 1528, 1542, 1566, 1641)

1572. Lee, G. H., and Van der Pyl, L. M., A bibliography on diaphragms and aneroids, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-180, 74 pp.

Authors believe that this bibliography includes all the available published data on diaphragms and aneroids up to the end of 1954. The search was conducted by the authors, and the annotation was performed by Lyman M. Van der Pyl, George H. Lee, and Lyman Cook. From authors' summary

1573. Gedeon, J., Development of pneumatical gages (in Hungarian), *Mérés és Automatika* **3,** 10, 301–311, Oct. 1955.

Author describes the action of the high-pressure pneumatic measuring instrument which is needed for fast and accurate measurements in mass production of parts. The modifications from the original Zisz model are described next. Theoretical considerations are given for accurate measurements with a pneumatic gage. Further development work is described and calibration experiments discussed. The final instrument, the R4 Pneumicro, is described and recommendations are made for further studies and development work. T. P. Torda, USA

1574. Dadson, R. S., A new method for the absolute measurement of high pressures, *Nature* **176,** 4474, 188–189, July 1955.

This is a brief note from the National Physical Laboratory, England, concerning present efforts in more accurately measuring the distortion occurring in piston cylinder assemblies used for measurement of high pressure (up to 3000 atm). The method is based on the assumption that, under certain conditions, two piston-cylinder assemblies of the same nominal dimensions, but of different materials having elastic contents which differ in a known ratio, will distort under applied pressure in proportion to one another throughout the pressure range. The main difficulty in the method lies in the construction of nearly identical highly accurate piston-cylinder assemblies. A complete description of the investigation will appear in a later publication.

R. J. Mindak, USA

1575. Haughton, C. B., Jr., and Gorton, R. E., Effect of mechanical vibration on the water flow through $^1/_4$-inch sharp-edged concentric ASME orifice, in a 1-inch pipe, ASME Ann. Meet., N. Y., Nov. 28–Dec. 3, 1954. Paper 54—A-113, 9 pp.

In the Fall of 1952, during the development of a new type flowmeter containing both variable-area and constant-area orifices the Fischer and Porter Company discovered that mechanical vibration was seriously affecting the performance of the constant-area orifice. Results of special vibration tests on this orifice led to the investigation presented here in which it was found that a severe vibration of 8 g produced a flow error of 55% at an orifice differential pressure of 0.95 in. of water. Shortly after these tests were completed, Pratt & Whitney became interested in the phenomena and set up a considerably improved test in which it was able to produce nearly pure transverse and axial vibrations of the flowing orifice and pipe. These tests demonstrated that axial vibrations produce far greater flow errors than transverse vibrations, and typical maximum error of 83% lower flow with a vibration of 5 g at 150 cps was measured. The Pratt & Whitney tests not only appear to confirm the Fischer and Porter tests but also show that the effects of g and frequency are considerable. A physical interpretation of what happens at the orifices is offered for consideration, and recommendations for future investigation are presented. The results reported in this paper should be of particular interest to users of very small orifices.

From authors' summary

1576. Junkins, J. H., Automatic-reading viscosimeter, *Rev. sci. Instrum.* **26,** 5, 467–470, May 1955.

An automatic-reading capillary-type viscosimeter has been constructed for measuring the relative viscosities of gases. The measurement of the viscosity is made in terms of the time required for the pressure exerted by a charge of gas in the capillary forechamber to decrease from one fixed value to a second fixed value, while the gas flows through the capillary into a constant pressure vessel. Computation of the absolute viscosity is based on the following relationship between the time t for gas A whose viscosity n is known, and the time for gas B whose viscosity is being determined:

$$\frac{t_{\text{gas } A}}{n_{\text{gas } A}} = \frac{t_{\text{gas } B}}{n_{\text{gas } B}}$$

Pneumatic and electrical instruments are employed to control

the temperature of the bath in which the capillary is suspended, maintain a constant pressure in the capillary exit vessel, and actuate the timer which measures the interval corresponding to a given pressure drop in the capillary forechamber. The performance of these instruments is completely automatic and primarily in this respect the apparatus described differs from manually operated expiration-type capillary viscosimeters. Small groups of consecutive measurements (e.g., five runs on dry nitrogen at 65.0 C) yield readings which deviate from the mean value by the order of 7 parts in 10,000. From author's summary

1577. **Ellis, S. C., Lanham, A. F., and Pankhurst, K. G. A., A rotational viscometer for surface films,** *J. sci. Instrum.* **32,** 2, 70–73, Feb. 1955.

The surface viscosity of monomolecular surface films (monolayers) spread on water has been measured with an instrument analogous to the coaxial cylinder (Couette) viscometer. A ring of platinum wire, coated with paraffin wax, is floated on the film-covered surface and rotated by a synchronous motor, transmission being through a torsion wire. The viscous drag imposed by the film on the rotating ring is calculated from the angular displacement between the ends of the torsion wire. Specimen results for serum albumen, showing its non-Newtonian behavior, are given. From authors' summary

1578. **Maron, S. H., Krieger, I. M., and Sisko, A. W., A capillary viscometer with continuously varying pressure head,** *J. appl. Phys.* **25,** 8, 971–976, Aug. 1954.

A new capillary viscometer has been designed to study the flow behavior of non-Newtonian fluids. The instrument is convenient, absolute, and accurate, and covers continuously a wide shearing stress range in a single determination. A falling mercury column forces the sample through the capillary. Measurement of the column height as a function of time gives both pressure drop and flow rate. Provision is also made for the use of driving fluids less dense than mercury for measurements at lower shearing stresses. The density of the sample need not be determined. Methods and illustrations are given of the application of the new instrument to the determination of the absolute viscosities of Newtonian liquids and the flow curves of non-Newtonian fluids. From authors' summary

1579. **Zysina-Molozhen, L. M., and Shapiro, I. G., Interferometric flow investigation of streamlines betweeen turbine blades** (in Russian), *Zh. tekh. Fiz.* **24,** 6, 978–989, June 1954.

Article presents results of comparative interferometric investigation of cascade arrangement of two turbine blade profiles. Analysis of photographic records reveals distribution of pressures and critical speeds on the surface and between the blades. This leads to determination of losses and blade efficiency. A. Petroff, USA

1580. **Batchelor, G. K., Binnie, A. M., and Phillips, O. M., The mean velocity of discrete particles in turbulent flow in a pipe,** *Proc. phys. Soc. Lond.* **68,** part 12, 432B, 1095–1104, 1955.

By clever deductions confirmed by careful experiments, authors have found a simple and accurate means of determining the rate of water flow in round pipes, applicable, for example, to large-scale hydraulic installations. The scheme is to insert into the pipe, one at a time, small spherical particles having the same density as water and to time their passage over a sufficiently long straight run of pipe. If one intuitively distrusts this method, he is in for a pleasant surprise. The paper therefore has intellectual as well as practical appeal. It is shown that the velocity of a material element of fluid in turbulent pipe flow averaged over a sufficient time is the same for all elements, since they can wander freely over the whole cross section, and is the same as the mean velocity as commonly defined. The same would be true of a solid sphere except for the fact that its finite size excludes it from a region near the wall. This effect is treated theoretically and tested experimentally. Consideration of the axial dispersion due to turbulence yields a dispersion coefficient which serves in a statistical treatment, along with universal flow parameters, to specify the number of observations needed for a given accuracy. G. B. Schubauer, USA

1581. **Seddon, J., and Spence, A., A wind-tunnel investigation of entry loss on propeller turbine installations. Parts I and II,** *Aero. Res. Counc. Lond. Rep. Mem.* no. 2894, 56 pp., Aug. 1948, published 1954.

Report is in two parts, following a general introduction. Part I describes wind-tunnel tests on (a) a series of models of annular entries, with and without propeller, in the 5-ft tunnel; (b) a set of large circular blade roots on a full-size nacelle in the 24-ft tunnel.

The models were based on two representative propeller turbine engines of different sizes. Various shapes and sizes of spinner and duct were tested, including vertical and sloped entries and elliptical and conical spinners. The work follows on from past tests, model and full-scale, on entries for radial air-cooled reciprocating engines.

The smaller engine tends to have the higher entry loss, owing to the blade roots being relatively thicker. In a typical case, under cruising conditions, the total entry loss on the model is 25% of free-stream dynamic head, of which 15% is caused by the blade roots. Scale effect is likely to be small. In these circumstances a large diameter spinner gives the best result. Slope entries are not recommended.

From a generalized analysis of the results, empirical rules are suggested for the estimation of spinner loss, duct loss, and blade-root loss, making up the total entry loss in flight. The additional duct loss which is usually present in ground running is also considered in general terms.

Part II describes wind-tunnel tests on models of a number of alternative ducted spinners for a typical engine, and for comparison, one annular entry similar to those tested in part I.

It is shown that the ducted spinners give 90 to 95% total head in cruising flight compared with about 75% for the annular entry. Most of the gain is in a reduction of blade-root loss from 17% total head to about 2%. The results are not sensitive to the shape of the blade-root fairing.

Low velocity must be maintained as far as possible, both in the spinner itself and in the rear duct. Expansion of the duct in the neighborhood of the leak should be avoided, however. The leak gap should be kept small, to minimize the extra flow taken through the spinner.

A short cowl version, in which the outer cowl of the spinner terminates just ahead of the propeller, is satisfactory for practical purposes, and has the advantage of being lighter in weight than a long cowl spinner with nose entry.

A detailed analysis of the loss is given, using methods evolved in part I. From authors' summary

1582. **Neuberger, W., Replacement of wind-tunnel measurements with special regard to measurements in free flight** (in German), *Z. Flugwiss.* **3,** 7, 216–235, July 1955.

An investigation in 1937 by A. Betz showed that the power of modern wind tunnels for measuring air reactions on aircraft models had increased tenfold in 12 years. Extrapolation to 1950 gave 10^5 hp which is, in fact, the power available for the high-speed wind tunnel in Modane, France (in service since

1951). A re-investigation by H. Slichting showed that a modern wind tunnel will require 10^6 hp in 1960.

Author thinks this evolution will lead to constructions too expensive to build and run. The possibility of making free-flight measurements should be investigated. This method could probably be made as accurate as measuring in a wind tunnel and does not imply such expensive facilities. Evaluation of results will be more extensive but this could be done on an electronic computer which is expensive but which could evaluate measurements from many different places.

Three current principal methods of measuring air reactions are described and compared: (1) Direct measurement by 6-component balance; (2) indirect by measuring pressures on surface of the body; and (3) indirect by recording motion of the body (free-flight measurement). Last method has been thoroughly investigated by means of motor analysis. Dynamical relations include effect of atmospheric wind.

Measuring equipment needed for the different methods is analyzed. For free-flight measurements, a stereophotographic method of measuring the motion of the aircraft is described in some detail.

In udging the advantages of the different methods, author stresses the inconvenience in not being able to get simultaneous Mach and Reynolds similarity and the difficulty of making corrections for turbulence in a wind tunnel. K. Flodin, Sweden

1583. Letko, W., and Williams, J. L., Experimental investigation at low speed of effects of fuselage cross section on static longitudinal and lateral stability characteristics of models having 0° and 45° sweptback surfaces, *NACA TN* 3551, 45 pp., Dec. 1955.

An experimental investigation at low speed was made to determine the effects of fuselage cross section on the static longitudinal and lateral stability characteristics of midwing airplane models having 0° and 45° sweptback surfaces. The results indicated that the main effects of fuselage cross section on the longitudinal and direction stability characteristics of the models at low angles of attack are caused by the direct contributions of the fuselage. At the high angles of attack, in addition to the direct contributions of the fuselage, wing-fuselage interference (sidewash) with the tail decreases the tail contributions to the directional stability. The configuration consisting of the wing and deep fuselage produced the most detrimental effect, and the configuration consisting of the wing and shallow fuselage was the least detrimental in this respect. For the complete configurations tested, fuselage cross section had little effect on the range of linearity of the curves of yawing moment against sideslip angle for the angles of attack and sideslip investigated. In general, the configuration with the deep fuselage had the poorest directional characteristics of the models investigated.

From authors' summary

1584. Morrow, J. D., Measurements of the effect of trailing-edge thickness on the zero-lift-drag of thin low-aspect-ratio wings, *NACA TN* 3550, 11 pp., Nov. 1955.

Results of an exploratory free-flight investigation at zero lift of several rocket-powered drag-research models having tapered 4%-thick wings are presented for a Mach number range of 0.7 to 1.6. Wings having an aspect ratio of 3.11 and trailing-edge thicknesses equal to 0, 1/3 maximum thickness, 2/3 maximum thickness, and the maximum thickness were tested. The sections were identical circular arcs back to the 40%-chord station. The remainder of the section was formed by drawing a tangent from the trailing edges of various thicknesses to the extended circular arc. The data obtained indicated that an increase in the ratio of trailing-edge thickness to maximum thickness caused a corresponding increase in wing drag coefficient throughout the Mach number range investigated, which was due to the increased area over which the base suction acts.

By use of the linear theory and base-pressure values measured on a 6%-thick blunt-trailing-edge wing, the calculated wing drag coefficients at Mach numbers from 1.3 to 1.5 compared favorably with the test result. Thus, the base pressure coefficient of blunt-base airfoil sections appears to be constant with base thickness in the range of thicknesses from 6 to 1.3% chord for Mach numbers of 1.3 to 1.5. From author's summary

Thermodynamics

(See also Revs. 1469, 1470, 1526, 1574, 1575, 1576, 1599, 1600, 1603, 1604, 1605, 1609, 1613, 1614, 1618, 1622, 1623, 1625, 1626, 1627, 1628, 1636, 1645, 1652)

1585. Bordoni, P. G., On the exact relation between the specific heats of an elastic solid, *J. rational Mech. Analysis* 4, 6, 975–981, Nov. 1955.

The same relation between the specific heats at constant volume and at constant pressure is shown analytically to apply to both fluids and solids. The common relation is obtained from the general theory of finite thermoelastic deformations without introducing any hypothesis on the type of strain energy and by considering only a certain invariance characterizing dependence of thermoelastic potential on temperature. J. F. Lee, USA

1586. Glansdorff, P., and Prigogine, I., On the differential properties of entropy production (in French), *Physica* **20,** 10, 773–780, Oct. 1954.

The time variation of the entropy production (per unit volume and unit time) is split into two parts. The first is related to the changes of the generalized forces with time and the second to the changes of the rates of irreversible processes. If some restrictive conditions are satisfied (linear phenomenological laws, Onsager's reciprocity relations, constancy of the phenomenological coefficients), both parts are equal and decrease in time.

Postulating *only* mechanical equilibrium and time-independent boundary conditions, authors show that the first part, related to the change of the generalized forces, is always negative or zero. The change of the generalized forces with time is, therefore, such that it lowers the value of the entropy production. Some consequences of this theorem are briefly discussed.

From authors' summary by L. M. Grossman, USA

1587. Krook, M., Dynamics of rarefied gases, *Phys. Rev.* (2) **99,** 6, 1896–1897 (Letters to the Editor), Sept. 1955.

1588. Hausen, H., Extrapolation of property diagram of steam with thermodynamic check (in German), *Brennstoff-Wärme-Kraft* **7,** 8, 341–345, Aug. 1955.

Author extrapolates from 300–400 bar by eye the t-curves in the p,pv-diagram (A) and also for the enthalpy h in the p,h-diagram (B), using the known curve between 200–300 bar. Thermodynamical formulas derived from first and second law are used as checks. Using first the relation for p const $(pv/T)_2 = (pv/T)_1 - p_1 \int^2 (\partial h/\partial p)_T dT/T^2$, the diagram (A) is corrected so that correspondence between the corrected diagram (A) and (B) is obtained. For the entropy, the formula $s_2 = s_1 + {}_1\int^2 (\partial h/\partial p)_p dT/T$ is used. From the resulting diagrams the following is obtained by 392 bar, 700 C: $v = 0.010 \text{m}^3/\text{kg}$, $h = 3676$ kJ/kg, $s = 6.377$ kJ/kgK, when h and s are taken as zero for water of

0 C and 0.006 bar. Reviewer has, in another paper not yet published, found for this point $v = 0.010$, $h = 3660$, $s = 6.37$.
M. Bäckström, Sweden

1589. **Bonilla, C. F., Wang, S. J., and Weiner, H., The viscosity of steam, heavy-water vapor, and argon at atmospheric pressure up to high temperatures,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-6, 4 pp.

This experimental study covers the determination of the viscosity of several important engineering materials at atmospheric pressure and high temperatures. The capillary used was made of 90% platinum and 10% iridium, having dimensions of 0.025-in. ID, 0.008-in. wall thickness, and length of 28.5 in. The capillary was coiled in seven turns around a cage. The results are presented in tables, and equations have been established to represent the data. Several charts are presented to indicate the deviation of the data. The results are of value to engineers in the design of certain types of apparatus. G. A. Hawkins, USA

1590. **Plewes, A. C., Jardine, D. A., and Butler, R. M., The integral heats of vaporization of alcohol-water mixtures,** *Canad. J. Technol.* **32,** 4, 133–145, July 1954.

Integral heats of vaporization at constant composition were measured for methanol, ethanol, and n-propanol-water mixtures at pressures of 760 and 300 mm. Enthalpy-concentration charts were prepared from these data for distillation studies. Liquid-vapor measurements for the three systems indicated considerable deviation from Raoult's law, yet integral heats of solution obtained from the enthalpy correlations were small. A qualitative explanation for the behavior of the liquid mixtures is offered.
From authors' summary

1591. **Kretschmar, G. G., The thermal expansion of some liquids of interest as rocket fuels,** *Jet Propulsion* **24,** 6, 379–381 (Technical Notes), Nov./Dec. 1954.

The volume temperature expansion coefficient has been determined for nitromethane and three dilutions of hydrazine, over the temperature range 0 to 75 C. Corrections for the expansion of the glass dilatometer were avoided by a preliminary volume calibration with mercury over the complete temperature range.
From author's summary

1592. **Thomson, W. R., Empirical equations for specific heat,** *Aircr. Engng.* **27,** 318, 251–253, Aug. 1955.

1593. **Mayers, M. A., Matiuk, A., and Baron, S., Combination gas turbine and steam,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-184, 10 pp. + 3 tables, 13 figs.

Authors discuss the development of two highly efficient coal-fired combination gas and steam turbine power plant cycles and the effects of a number of cycle variables on the plant efficiencies. Some of the engineering problems of these cycles are discussed. Investment cost estimates and evaluations are presented showing the annual production savings attainable with the combination cycles. The results of these studies are impressive and show that these cycles hold a great deal of promise as an economical arrangement for power generation.
From authors' summary

1594. **Alpert, S., Grey, R. E., and Drake, D. D., Application of internal liquid cooling to gas turbine rotors,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-202, 21 pp.

Application of internal liquid cooling to the rotor and blades of a gas turbine allows the inlet gas temperature to be increased and alloys low in strategic materials to be used. A small gas turbine was designed for a gas temperature of 1750 F with lean alloy rotor materials. The design featured closed system cooling with interchangeable blades. Cold air tests, hot tests, and endurance tests totaling 100 hr, have been run on a single-stage test turbine. Aerodynamic and thermodynamic data correlated well with expected values and the mechanical performance was satisfactory.
From authors' summary

1595. **Morabito, J. J., and Shannon, R. H., Test of a once-through steam generator with a liquid metal as a heat source,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-189, 31 pp. + 21 figs.

Paper covers the basic concepts and factors involved in the design and testing of a once-through steam generator utilizing a liquid metal as a heat source. The primary purpose of the test is to prove the feasibility of this type of unit for use in a nuclear power plant. From authors' summary

1596. **Birmann, R., Aerothermodynamic considerations involved in turbocharging four- and two-cycle Diesel engines,** *Trans. ASME* **78,** 1, 171–183, Jan. 1956.

The turbocharged engine is, in effect, a compound engine in which the engine itself handles the high-pressure end of both compression and expansion, while the exhaust-turbine-driven compressor takes care of the low-pressure end. The turbocharger pressure ratio determines the division of work between high-pressure and low-pressure components, and the value of this pressure ratio governs, to a large extent, the values of all engine performance variables. In this paper the interrelationships between all of these variables are discussed, and curves are presented giving, numerically, the interdependence, one on the other, of turbocharger pressure ratio, thermal loading, specific fuel consumption, and turbocharger combined efficiency, with and without intercooling. The aerothermodynamic situation of the two-cycle engine is treated separately, with emphasis on the all-important flow-handling ability of the engine. This ability is defined by the concept of the "gas change process merit ratio," and, by means of a series of curves, the specific engine air flow is given for different merit ratios and different turbocharger combined efficiencies for a range of turbocharger pressure ratios. Finally, for both two-cycle and four-cycle engines, the importance of good matching between engine air consumption characteristics and turbocharger air delivery characteristics is discussed, and as a criterion for this matching the engine operating line, as it appears superimposed on the compressor characteristic curves, is used.
From author's summary

1597. **London, A. L., The supercharged-and-intercooled free-piston-and-turbine compound engine. A cycle analysis,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-147, 8 pp. + 4 tables, 5 figs.

Heat and Mass Transfer

(See also Revs. 1363, 1477, 1512, 1585, 1592, 1595, 1596, 1633, 1636, 1639, 1663)

1598. **Ashton, M. D., Perkins, R. F., and Savage, L. H. W., Ingot heat conservation. The cooling of rimming-steel ingots between teeming and stripping,** *J. Iron Steel Inst. Lond.* **181,** 4, 303–312, Dec. 1955.

A method is described of reducing fuel consumption in soaking pits, increasing pit availability, and increasing mold life by stripping all ingots at the earliest practicable time. This method is based on a series of investigations which have been carried out on 8- and 15-ton rimming-steel ingots.
From authors' summary by G. Horvay, USA

1599. Winter, F. R., and Drickamer, H. G., Thermal diffusion in liquid metals, *J. phys. Chem.* **59,** 12, 1229–1230, Dec. 1955.

Thermal diffusion measurements have been made on a series of liquid metal mixtures. The results are totally consistent with a previously presented theory. The separations depend on the "activation energy density" $\Delta U \neq /V$ of the components. It is also shown that the quantity $X(\partial\mu/\partial X)$ in the denominator is important for quantitative prediction of the separation. Results are also presented and discussed for three ternary mixtures.

From authors' summary

1600. Somers, E. V., High-speed guarded-hot-plate apparatus for thermal conductivity of thermal insulation, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-129, 6 pp. + 4 tables, 4 figs.

This paper reports what appears to be an important advance in guarded hot-plate technique. The inconveniently long time, approximately 75 hr, required for equilibrium on standard hot plates was greatly reduced by Herman, Oswald, and others through the use of transient power excesses in recent work. The present paper further reduces this time, by combining the transient-power principle with a plate of very low thermal capacity, to less than twenty minutes in low thermal capacity materials.

The work is supported by analytical examination of the physical factors and by some test results with the new apparatus on light materials. F. C. Hooper, Canada

1601. Landis, F., and Tearnen, J. O., Graphical methods for the temperature response of bodies in a varying temperature medium, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-141, 15 pp. + 3 figs.

Paper is a contribution to practical solution of the important problems of heat conduction in bodies surrounded by a time-varying temperature field. The body is assumed at a uniform initial temperature, the heat-transfer coefficient at its surface remains constant, and the bath is considered as a series of small stepwise temperature changes.

Theoretical deductions are based upon Duhamel's superposition formula and refer to an infinite slab, but the method can be readily extended to cylinders, spheres, and parallelepipeds. This is also indicated in the paper.

All calculations are reduced to a summation of mutually similar terms and each of them is graphically tractable with the various charts used in the case of a constant-temperature medium. The method is illustrated by a practical example of cooling a thick glass plate by blowing air over its surface.

Reviewer recommends the paper as a valuable expedient for graphical determination of temperature fields in bodies submerged in media with time-varying temperatures.

V. Vodička, Czechoslovakia

1602. Codegone, C., Methods of measurement and determinations of heat conductivity in building materials (in Italian), *Termotecnica* **9,** 10, 474–490, Oct. 1955.

1603. Pascal, A., Measurement of heat conduction coefficients of building materials (in French), *Ann. Inst. tech. Bât. Trav. publics* **8,** 90, 583–598, June 1955.

A 50-cm square guarded hot plate of the same general type and characteristics as the ASTM approved type is described in detail. It is of the horizontal, two-specimen form.

Some detailed examinations of edge loss and other errors are reported, and it is recommended that samples up to 17.5 cm can be used without edge insulation. This is considerably thicker than the recommendations of most other workers in this field for hot plates of similar proportions. F. C. Hooper, Canada

1604. Jaeger, J. C., Conduction of heat in a solid in contact with a thin layer of a good conductor, *Quart. J. Mech. appl. Math.* **8,** part I, 101–106, Mar. 1955.

The problem of a solid immersed in a poor conductor is studied, and practical application to thermocouple usage is suggested. Because of low thermal conductivity of the medium around the thermocouple, an error is expected, particularly in the transient response, for thermocouples where conduction along the wire is significant.

Author derives general equation and gives specific numerical solutions. It is pointed out that 28-gage copper wire between planes 10 cm apart and immersed in a substance with thermal properties of water will give rise to a significant error in temperature.

Reviewer believes paper will be useful to workers involved in transient temperature measurements in materials of low thermal conductivity. R. H. Eustis, USA

1605. Nicolescu, M., Iterated heat equation (in Rumanian), *Studii si cercetari Matematice* **5,** 3/4, July-Dec. 1954.

The real object of this article is a study of the solutions to the following equation:

$$\Omega^{(p)}u = [\Delta - (\partial/\partial t)]^{(p)}u = 0$$

where

$$\Omega = \Delta - (\partial/\partial t)$$

as well as the solution of several related boundary problems.

For brevity, author refers to the entire solution of

$$\Omega u = \Delta u - (\partial u/\partial t) = 0$$

as a thermal function, which is continuous in certain known regions, and calls the known solutions to above equation polythermal functions of the p^{th} order.

In the first chapter, it is shown that all polythermal functions of the p^{th} order in the vectorial domain are analytical.

In chaps. 2 and 3, using Poisson's formula, extensions are established for polythermal functions of p^{th} order.

In the last paragraph of chap. 3, author introduced parabolically analytic functions which are defined as all functions $u(x,t)$ in the interval $0 < t < 1/4k$ if they can be written in the form

$$u(x,t) = \sum_{n=0}^{\infty} (t^n/n!)u_n(x,t)$$

Chap. 4 is devoted to boundary problems for the bounded intervals. Author then solves the problem: Find a polythermal function of the second order on the interior of a region D. It is shown that the solution of this problem comes about from the solution of four linear integral equations of the Volterra type. The statement of the problem requires a study of a special bithermal function having a certain type of discontinuity on one of the bounding curves of D. Author calls this function "the bithermal potential" because of the part it plays in the solution of this problem.

In chap. 5, author demonstrates that if the function u possesses continuous partial derivatives necessary for the function of the differential operator $\Omega^p u$ and if these derivatives are selectively algebraically increased, then the integral obtained from Poisson integral by a change of variables must be derivable at the point $h = 0$, when the integral is considered a function of h, p.

The final chapter of this paper is devoted primarily to a study of the properties of polythermal functions defined for the interval $0 \leqq t \leqq t_1$. The basic tool throughout this study is a theorem of

the mean which satisfies all polythermal functions subjected to certain conditions of regularity as $x \to \infty$.

The last paragraph of this chapter is concerned with subthermal functions (of order 1,2,3) defined in the interval $[0, t]$. Author indicates the difference between his definitions for subthermal functions and those defined by I. G. Petrowski in 1935.

N. Matusewicz, USA

1606. **Low, G. M., Reversal of skin-friction and heat-transfer trends in high-speed laminar flows,** *J. aero. Sci.* **22,** 10, 721–722 (Reader's Forum), Oct. 1955.

Conditions of wall temperature and Mach number for which the leading edge values of skin friction drag or heat transfer on an airfoil decrease at the same rate as on a flat plate are defined as skin friction or heat-transfer reversal conditions. Author's compressible laminar boundary-layer solutions with heat transfer and small pressure gradient [*NACA TN* 3028; AMR **7,** Rev. 1191] are employed to indicate Mach numbers and wall temperatures for which reversal occurs.

Basic assumptions are viscosity proportional to temperature, isothermal surfaces, Prandtl number of 0.72. External flow characteristics and boundary-layer variables are expanded in Taylor series about their values at the leading edge. Only the linear terms in these series are retained. R. J. Kirby, USA

1607. **Roy, M., Aerodynamic heating versus speed; thermodynamic aspects of the struggle** (in French and English), *ONERA Publ.* 77, 51 pp., 1955.

Paper concerns itself with a brief discussion of the following topics: The tolerable temperatures, aerodynamic heating, thermodynamic behavior of boundary layers, internal and external heat exchange of supersonic aircraft, effects of sustained flight time, external insulation and thermal partitioning, fuel protection and possible use of fuel as a coolant, forced cooling systems, and effect of aerodynamic heating on jet-engine operation. Text is adequately diagrammed. Eighteen bibliographical references are cited. (Graduate students note!! The French and English pages and diagrams are opposite, making this paper a valuable aid to amplifying a technical French vocabulary.)

C. R. Mischke, USA

1608. **Backus, G. E., On the application of eigenfunction expansions to the problem of the thermal instability of a fluid sphere heated within,** *Phil. Mag.* (7), **46,** 383, 1310–1327, Dec. 1955.

The onset of convection in a fluid sphere heated from within is described by an eigenvalue problem. The lowest eigenvalues (Rayleigh numbers) have been previously determined by variational techniques. There are two disadvantages in estimating the eigenvalues in this manner: (1) only upper bounds can be obtained, so that in principle the true values cannot be bracketed; and (2) the higher eigenvalues can only be obtained from lower ones which must, as a consequence, be known with extreme accuracy.

In present paper the above-cited disadvantages are overcome by expanding the eigenfunctions (convective velocity and temperature) in spherical Bessel functions. This procedure is more laborious, but, for example, any eigenvalue can be computed independently of the others. A table of a number of eigenvalues is presented and a comparison of the lowest eigenvalues computed both ways is made.

The class of problems which should be amenable to this method is delineated and, in particular, it is shown that the classical Rayleigh-Bénard problem of convection in a horizontal layer of fluid is in this class. S. Ostrach, USA

1609. **Sparrow, E. M., and Gregg, J. L., Laminar-free convection heat transfer from the outer surface of a vertical circular cylinder,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-25, 6 pp.

The partial differential equations expressing conservation of mass, momentum, and energy for laminar, free convection in a boundary layer on the outer surface of a vertical circular cylinder are transformed into a form in which it is convenient to expand the solution in a power series in one variable with coefficients which are functions of the other. The mathematical validity of the procedure is not investigated, but results based on the truncated series are compared with those for a flat plate. Comparison is also made with analysis based on pure conduction through an equivalent (fictitious) layer of stagnant fluid.

H. D. Block, USA

1610. **Jones, C. D., and Masson, D. J., An interferometric study of free-convection heat transfer from enclosed isothermal surfaces,** *Trans. ASME* **77,** 8, 1275–1281, Nov. 1955.

Variations of point and average heat-transfer coefficients with spacing between surface and enclosure are presented for (a) horizontal isothermal cylinders near plane partial enclosures and (b) vertical isothermal prisms of small height in a duct.

The results for enclosed bodies differed significantly from those for unconfined bodies. Some inconsistencies of present results with similar studies previously made by others are discussed.

S. Ostrach, USA

1611. **Sparrow, E. M., Laminar free convection on a vertical plate with prescribed nonuniform wall heat flux on prescribed nonuniform wall temperature,** *NACA TN* 3508, 34 pp., July 1955.

Analysis is made for laminar free convection on vertical plate with either prescribed nonuniform heat-transfer rate or prescribed nonuniform temperature at surface. The von Karman-Pohlhausen method is employed to solve boundary-layer equations. Density is assumed variable in buoyancy terms; however, in all other terms all fluid properties are considered constant. Viscous dissipation is neglected. Results are given graphically for several special cases. M. W. Rubesin, USA

1612. **Fillipov, L. P., Effect on the heat-transfer process of radiation and absorption by the medium** (in Russian), *Vestnick, Moscow Univ.* **1,** 2, 51–57, 1954 (translated from Russian by M. D. Friedman, 572 California St., Newtonville, Mass. 9 pp.).

A steady-state solution is obtained for an absorbing medium between plane, partially reflecting walls. Solution is obtained under physical assumptions and boundary conditions permitting linear approximation of conductive and radiative properties of the fluid; namely, small temperature difference between the walls, radiative coefficients independent of temperature, thermal conductivity such that temperature distribution departs only slightly from linear. Deviation of temperature gradient from linearity is evaluated for weakly and strongly absorbing media. Radiative influence on local and total heat flow is estimated. Treatment has interest because approach of this fundamental nature has not been sufficiently exploited for use by engineers in this country. M. Gilbert, USA

1613. **Hooper, F. C., The possibility of complete solar heating of Canadian buildings,** *Engng. J. Montreal* **38,** 11, 1501–1506, Nov. 1955.

Author discusses his conclusion that solar house heating in southern Ontario has good prospects for the future. System proposed involves storage of summer solar heat for winter use,

using system with relatively small solar collector and large storage system (about 600 sq ft of collector area with 75 U.S. gallons of water per sq ft of collector area for heat storage for a small, well-insulated house), and with no auxiliary heater. Author believes this system has advantage over designs proposed for Boston and Denver [Whillier, A., and Löf, G. O. G., respectively, *Proc. World Symposium on Applied Solar Energy*, Phoenix, Ariz., Nov. 1955] which provide for only partial solar heating during the winter, which have about the same collector size but only about one thirtieth of the storage capacity, and which do have conventional heating systems for auxiliary heating when needed. Economic success of the design hinges largely upon the cost of the large storage system and the finding, for example, of suitable phase-change storage materials (other than Glauber's salt and disodium phosphate dodecahydrate, which are not satisfactory) might well swing the balance decidedly in favor of storage of summer heat for winter use. Reviewer believes the design has much merit and justifies full-scale testing.

A. Whillier, South Africa

1614. Baum, V. A., The use of solar energy, *J. sci. indust. Res. India* **14A,** 8, 365–373, Aug. 1955.

Author describes developments in the USSR, including flat-plate collectors; cooking devices; and reflecting-type collectors used for multiple-effect distillation, for making ice with absorption refrigerators, and for power generation. Only the southern arid regions of the USSR are economically suitable.

There is almost no market for flat-plate collectors, and author feels that only the reflecting-type, for steam generation, has any economic advantage. [This is in contrast to the United States, where the flat-plate collector has far greater economic advantage. Reviewer believes this difference is due to the vast difference in living standards. In the United States, almost every new house has central heating and a hot-water supply, whereas in the USSR few houses have these facilities. There are, however, communal bath facilities, communal cold storage (if any), etc., which would perhaps justify the employment of engineers to maintain the complex reflecting equipment. The average householder in the USSR cannot afford a heating system and, hence, there is no market for flat-plate collectors.]

Figures given for the output of the various types of solar collection equipment are in agreement with results obtained in the United States and elsewhere.

A. Whillier, South Africa

1615. Kayan, C. F., Electrical analog analysis of a cooled structure complex, *Heat. Pip. Air Condit.* **27,** 11, 143–148, Nov. 1955.

Structures and buildings maintained below atmospheric temperatures represent a composite arrangement in which the inward flow of heat and heat from sources inside the conditioned space must ultimately be absorbed by some cooling medium within same space. Mathematical treatment of such problems, particularly if transient heat transfer enters, is quite difficult and laborious. By employing the electrical analogy technic, with the resistance concept extended to include even refrigerating machine performance, author has given the general outline of how to predict performance under varying operating conditions. An illustrative example complete with diagrammed heat transfer and working condition parameters is added. There are references to former papers of author, where some of the used concepts are defined and explained. This paper is another example of how well the electrical analogy method is adaptable for solving complex heat-transfer problems in an easy way.

C. E. Lenngren, Sweden

1616. Rizika, J. W., Liquid droplet heating and evaporation in a high-temperature gas stream, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-154, 16 pp. + 7 figs.

Author calculates duct length for vaporizing injected liquids, assuming two distinct stages: heating of liquid to boiling point without vaporization, vaporization without further changes of liquid temperature. Further assumptions are: liquid conductivity infinite, heat transfer proportional to temperature difference, atomization follows Nukiyama-Tanasawa equation. Author accounts for fall in gas temperature, inequality of droplet and gas velocities. No experimental confirmation is cited. Validity of assumptions is not discussed.

Reviewer believes approximations are too sweeping to justify calculation of such detail. D. B. Spalding, England

1617. Sabersky, R. H., and Mulligan, H. E., On the relationship between fluid friction and heat transfer in nucleate boiling, *Jet Propulsion* **25,** 1, 9–12, Jan. 1955.

Von Karman [*Trans. ASME* **61,** 705–10, 1939] has shown that the friction and heat-transfer coefficients on the basis of Reynolds' analogy are related by the equation $C_F/2 = C_H$ when the Prandtl number is unity. The same relation is assumed for turbulent flow without a laminar boundary layer. The authors conducted experiments to determine if any relationship exists between the friction and heat-transfer coefficients in the region of nucleate boiling with forced convection.

Tests were made with distilled water flowing through a 3/8-in. diam stainless-steel tube, the tube itself serving as an electrical resistance heater. Authors' test results confirm the above equation to a first approximation. Reviewer believes that further research is needed to show that the above equation is independent of other parameters. The present test data suggest a definite pressure effect, and some uncertainty exists in the temperature predicted at the inside surface of the tube. Reviewer agrees with the authors that the first-order effect of the bubbles is a hydrodynamic one, leading to an increase in heat exchange as well as momentum exchange. R. L. Scorah, USA

1618. Lane, A. M., and Stern, S., Application of superheated vapor atmospheres to drying, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-218, 6 pp. + 10 figs.

1619. Amundson, N. R., Solid-fluid interactions in fixed and moving beds. Fixed beds with small particles. Fixed beds with large particles, *Indust. Engng. Chem.* **48,** 1, 26–43, Jan. 1956.

Expressions are derived for the temperatures of fluid and solid in a cylindrical fixed bed with turbulent flow, under the restriction that the rate of production of heat is determined uniquely as a linear function of the temperature. Reviewer has not encountered any reactions satisfying this restriction, but the results are valuable when heat transfer is important and no significant amount of heat is produced by chemical reaction. A pebble heater initially at uniform temperature, for example, would be adequately described. J. Beek, Jr., USA

1620. Siegmund, C. W., Munro, W. D., and Amundson, N. R., Solid-fluid interactions in fixed and moving beds. Two problems on moving beds, *Indust. Engng. Chem.* **48,** 1, 43–50, Jan. 1956.

Expressions for the temperatures of fluid and solid in a cylindrical countercurrent moving bed are worked out, taking account of transfer of heat through the walls of the cylinder. The restrictions imposed are the same as in a prior paper (see preceding review), excluding the reactions ordinarily encountered

The results would be applicable to heating or cooling a fluid in such a system. A method for numerical solution for nonlinear cases is indicated. Reviewer believes that this is especially useful, since all practical cases involving reactions that produce a significant amount of heat must be treated numerically.

J. Beek, Jr., USA

1621. **Datsko, S. C., Metal transport in liquid water systems and effect on heat transfer,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-183, 8 pp. + 3 tables, 9 figs.

A series of tests was conducted to determine the effects, on heat transfer, caused by the corrosion by water of zirconium tubes and by the deposition of corrosion products of a stainless-steel system on the zirconium-tube walls. Tests were conducted at water velocities from 2 to 30 fps and heat-flow rates from 30,000 to 500,000 Btu/hr-sq ft of surface in single-tube, countercurrent, water-to-water heat exchangers. The heating-water temperature ranged between 500 and 620 F and the cooling-water temperature between 450 and 500 F. Several water treatments, such as deionization and oxygen or hydrogen additions, were investigated. Results of these tests showed that: (a) Corrosion of zirconium at these temperatures was not sufficient to cause measurable increases in heat-transfer resistance; (b) with oxygen concentrations up to 2 cc per liter of water, no measurable deposition occurred on the tubes at any of the combinations of low water velocities and low heat fluxes or high water velocities and high heat fluxes, even though no removal of corrosion products was attempted; (c) at oxygen levels between 100 and 1000 cc per liter, sufficient deposition of stainless-steel corrosion products occurred on the cooled side of the zirconium tube to decrease heat-transfer coefficients approximately 50%; such deposits could be prevented or removed by continuous operation of an ion exchanger; (d) there were indications that either hydrogen additions or cold-water washing would also remove the deposits; (e) deposition appeared to occur only on regions of high heat-transfer rates and was associated with pH between 5 and 7.

From author's summary

1622. **Harper, D. B., Seal leakage in the rotary regenerator and its effect on rotary-regenerator design for gas turbine,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-109, 11 pp. + 13 figs.

The application of rotary regenerators to gas turbines is considered in some detail, particular emphasis being given to the question of reducing leakage by balancing pressures. Interconnection between the seal shoes across the rotor is shown to be particularly advantageous, while controlling the pressure of the seal shoes on the rotor from the pressure under the seals is also discussed.

Experimental verification is given and a design study for a regenerator for a 4000-hp gas turbine summarized. The paper contains much valuable information for those interested in this subject.

G. G. Thurlow, England

1623. **Pohl, O., Experimental results obtained to date with air condensers** (in Hungarian), *Magyar Energiagazdaság* **8**, 10, 361–369, Oct. 1955.

Experiments carried out to date have shown that the Heller-type steam condenser which uses air as the coolant is suitable for application to larger power-station applications as far as the cooling tower, the mixing condenser, and reliability are concerned. Although the initial cost of the condensing equipment is greater than the conventional ones, this system is the only good one in regions with water shortage. Regarding total cost, the Heller-type system may be considerably more advantageous in certain cases (the initial costs of equipment for obtaining water and filtering it is not necessary). There is a possibility that the new system will be competitive with the old-type (water-cooled) cooling tower, even if placed in regions where water is not too scarce.

From author's summary by T. P. Torda, USA

1624. **Vasenin, F. I., Thermoelectric properties of systems of bismuth-tellurium alloys** (in Russian), *Zh. tekh. Fiz.* **25**, 3, 397–401, 1955 (translated from Russian by M. D. Friedman, 572 California St., Newtonville 60, Mass., 5 pp.).

Some forty alloys were prepared. Pressed samples were also used and annealed. The compound Bi_2Te_3 is p-type with a thermoelectric power of 210 microvolt/°C. Excess Te produces n-type samples, which are influenced by annealing, maximum thermoelectric power being around −170 microvolt/°C. The room temperature electric conductivity is rather high, about 300 to 470 ohm^{-1} cm^{-1} for the compound, and around 1000 for the n-type alloy, depending upon annealing.

M. Telkes, USA

1625. **Rama, S., Relative errors of thermocouples used for measurement of surface temperatures** (in Italian), *Termotecnica* 9, Suppl. "Ricerche no. 5," 11, 251–560, 1955.

Mathematical investigation of thermocouple errors when used for measurement of surface temperatures of solids. Problem is considered one of heat conduction, and calculations are boundary-value problems for special geometric configurations taking account of thermometric properties of both solid and thermocouple. Methods needed for calculation are indicated, but example of extent of errors possible for extreme cases is not given. Treatment is otherwise comprehensive.

J. M. DallaValla, USA

1626. **Rabinowicz, J., Jessey, M. E., and Bartsch, C. A., Resistance thermometer for transient high-temperature studies,** *J. appl. Phys.* **27**, 1, 97–98, Jan. 1956.

1627. **Schneider, R., Measurement of small temperature differences with the aid of temperature-sensitive electrical resistance bridges** (in German), *ATM* 212/3, 237, 217–220, Oct. 1955.

Combustion

(See also Rev. 1585)

1628. **Penner, S. S., Introduction to the study of chemical reactions in flow systems (AGARDograph no. 7),** London, Butterworths Scientific Publications, 1955, viii + 86 pp.

The aims of this book are stated to be: (1) To present an adequate summary of the principles of classical chemical kinetics which is intelligible to investigators without previous training in chemical kinetics; (2) to provide the necessary basic material for intelligent formulation of flow problems with chemical reactions.

The initial chapter of the book is written to accomplish the first of these. The summary is adequate and clear, but it is highly concise, as any presentation of chemical kinetics in one chapter must be. Since copious mathematical notations are also used, the chapter is one which will require very intensive reading by one not familiar with the field.

Later chapters deal with conservation laws and transport properties in reacting mixtures, with the analysis of reaction and with heterogeneous chemical reactions. The latter term is not used in the ordinary sense of chemical reaction at a phase boundary, but includes rather heterogeneous systems in which chemical reaction is occurring. In particular, the effects of surfaces on explosion limits and the burning of single drops of fuel are discussed.

As a simple example of chemical reaction in a flow system, the problem of chemical reactions during adiabatic flow through a de Laval nozzle is analyzed in some detail. This includes relatively rapid reactions (near-equilibrium conditions) and very slow ones (near-frozen conditions).

Some selected references to experimental studies are included in the bibliography, but in general the approach is theoretical and highly mathematical rather than experimental and descriptive. R. C. Anderson, USA

1629. Mullins, B. P., Spontaneous ignition of liquid fuels, London, Butterworths Scientific Publications, 1955, xi + 117 pp.

Book is an effective review of present knowledge of ignition phenomena and a summary of experimental information published in the twentieth century.

Due emphasis is placed upon the fact that the spontaneous ignition temperature of a fuel is not an absolute quantity and must be interpreted in terms of the equipment and method used to measure this temperature. The important test methods—the Moore and Jentzsch heated crucibles, furnace and bomb methods, use of heated surfaces, adiabatic compression, flow methods, and others—are clearly described, and a highly condensed but comprehensive summary of ignition data on individual compounds is given. Effects of various fuel additives are also summarized.

The importance of ignition delay is also emphasized and the factors influencing curves of ignition temperature versus delay are discussed.

A concise report on theoretical concepts of the kinetics and mechanism of the ignition process is given. This is, of necessity, largely qualitative because of the relatively undeveloped state of this phase of ignition studies. The applicability of ignition data to various practical systems and engine processes is also discussed.

The bibliography on ignition phenomena is an excellent one. R. C. Anderson, USA

1630. Mecklenborg, K. T., Spontaneous ignition studies relating to lubricants of reduced flammability, *NACA TN* 3560, 17 pp., Jan. 1956.

Increased interest in the use of diesters as lubricants has suggested a study of the influence of the structure of the alcohol moiety on the spontaneous ignition temperature of a series of dialkyl azelates. The determination of spontaneous ignition temperatures has been extended to a number of representative organic phosphorus esters with predominantly aliphatic structures. Determination of the change in spontaneous ignition temperature with composition has been carried out with hydrogenated polyisobutylene in combination with a siloxane and with a phosphonate. The synthesis of polyisobutylene in the lubricant molecular weight range has been improved in yield.

From author's summary

1631. John, R. R., Wilson, E. S., and Summerfield, M., Studies of the mechanism of flame stabilization by a spectral intensity method, *Jet Propulsion* **25,** 10, 535–537, Oct. 1955.

Studies of bluff-body stabilized flames at several laboratories indicate transition at Reynolds number of about 10^4 from predominantly laminar to predominantly turbulent transport of atomic and molecular species across the boundary of the recirculation zone. Present authors support this view with nice technique for determining local fuel-air ratio. Method uses photomultiplier tube to detect radiation of CH and C_2, with filters for bands 100A wide centered at 4280A and 5080A. Calibration with a nearly laminar portion of flame at several fuel-air ratios related CH/C_2 intensity ratio to fuel-air ratio to make possible determinations of local fuel-air ratio in recirculation zone and its boundaries. Fuel-air ratios so measured support postulate of selective diffusion and laminar transport at low Reynolds number.

W. P. Jensen, USA

1632. Goodger, E. M., The combustion characteristics of a cyclindrical-rod burner system, *Coll. Aero. Cranfield Rep.* no. 90, 18 pp. + 21 figs., June 1955.

Report discusses the potentialities of vertically placed, circular rods as a flameholder. Kerosene is used as a fuel which is made to flow over the rod surface, and flameholding characteristics, such as surface roughness and rod diameter, are considered. Rods are placed in different ways, such as singly or in multiple rows. A wealth of data is cited, but unfortunately comparison with other methods of flameholding is difficult because the data are not presented in a conventional manner. Some interesting suggestions for future study are included.

A. B. Cambel, USA

1633. Markstein, G. H., and Schwartz, D., Interaction between pressure waves and flame fronts, *Jet Propulsion* **25,** 4, 174–176, Apr. 1955.

1634. Simon, D. M., and Wagner, P., Characterization of turbulent combustion by flame space and space heating rates, *Indust. Engng. Chem.* **48,** 1, 129–133, Jan. 1956.

Turbulent premixed propane-oxygen-nitrogen flames are stabilized above pipes of various sizes by use of an annular pilot flame. Long-exposure photographs of the flame brush are measured, and "flame volumes" are thereby obtained. A parameter Z is defined as the volumetric flow rate divided by the flame volume; this parameter, essentially the reciprocal of the residence time in the flame brush, is proposed as useful for characterizing turbulent flames. Data show that Z is nearly independent of flow velocity and approximately proportional to s_L/d, where s_L is the laminar burning velocity and d the burner tube diameter. This finding is compared with predictions of folded-flame and extended reaction zone theories of turbulent flame propagation, but no firm conclusions are reached. R. Friedman, USA

1635. Reynst, F. H., Pulsating, pressure generating combustion systems for gas turbines, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-56, 12 pp. + 11 figs.

Author proposes using the constant pressure combustion process as a means for improving gas turbine cycle efficiency. Some of the proposals for accomplishing combustion by this mode are novel, but as yet unproven. E. S. Starkman, USA

1636. Strong, H. M., and Bundy, F. P., Measurement of temperatures in flames of complex structure by resonance line radiation. I. General theory and application to sodium line reversal methods; II. Sodium line reversal by high-resolution spectroscopy; III. From absolute intensity measurements at high resolution, *J. appl. Phys.* **25,** 12, 1521–1537, Dec. 1954.

General methods have been developed for measuring temperatures directly from the complex flame structure, with high accuracy. Methods described can be used for determination of velocities and pressures in the exhaust gases of the rocket.

Part I describes how the theory of sodium-line intensity contours may be applied to analyzing sodium-line reversal results in complex flame structure.

Part II describes: (1) Temperature measurement in nonluminous complex flames using a modified sodium-line reversal method, i.e., a sodium vapor lamp shining through a rotating spiral disk to produce a background of graded brightness, with

an interferometer to give highly resolved D lines; (2) a method for determining optical depths of complex flames by comparing sodium vapor lamp intensity alone with intensity after transmission through flame; (3) an example illustrating (1) and (2). The accuracy estimated at 2450 K was $\pm 3\%$.

Part III describes: (1) The determination of effective temperature in a complex flame involving evaluation of intensities and emissivities; (2) an analysis of emissivity determination by comparing mirrored and unmirrored brightness of the flame; (3) a method of calibrating photographic plates for use with interferometer; (4) a test of the applicability of method to zoned flames; 5) methods of introducing sodium salts into the flame.

The modified sodium-line reversal can be used only for nonluminous flames; the method of part III seems to have wider application and greater accuracy.

Authors' descriptions of the practical limitations are realistic. However, their observation that the temperature limits of the comparison light are no longer limits for actual flame-temperature measurements is encouraging. It is suggested the referenced articles be tackled before this series is studied.

A. C. Tobey, USA

1637. Sutherland, G. S., Mahaffy, D. A., and Summerfield, M., Experimental flame temperatures of an ammonium perchlorate solid propellant, *Jet Propulsion* **25,** 10, 537–539, Oct. 1955.

Authors used brightness emissivity method for measuring flame temperature at burning propellant surface by comparison of Na D line emissivity with tungsten filament. Luminosity was increased by 0.1% NaCl admixed with propellant. Photomultiplier tube and optical system enabled concentrating on an area of $50 \times 200\ \mu$ in the flame. Results showed measured flame temperatures were always lower than theoretical; and those of propellants with coarse (150–200 μ) oxidizer particles were lower than those with fine (5–10μ) particles. Studies of pressure and other influences are under way. W. P. Jensen, USA

Acoustics

(See also Rev. 1657)

1638. Mason, W. P., Effect of dislocations on ultrasonic wave attenuation in metals, *Bell Syst. tech. J.* **34,** 5, 903–942, Sept. 1955.

Author calculates internal friction of metals produced by motion of dislocations from one position of minimum energy in the lattice to another. Two forces are considered: (a) the Peierls force, which is a periodic function of the position of the dislocation in the crystal lattice; (b) the line tension in the dislocation. The dislocations are assumed to be pinned by impurity atoms at points about 4×10^{-4} cm apart. At low temperatures and low stresses this pinning is rigid. On either side of the linear equilibrium configuration of the dislocation are trapezoidal equilibrium configurations, in which the middle portion of the dislocation has moved by one atomic spacing. The side links of the trapezoid are many atoms long, but short in comparison with the whole dislocation loop. The activation energy U required to pass from one equilibrium position to another is calculated. If the frequency of vibration of a loop in its potential well is γ (about 10^{10} sec^{-1}), there is a maximum absorption of sound waves of frequency $\gamma \exp(-U/kT)$. On this basis, the low-temperature absorption maxima observed by Bordoni [AMR **7,** Rev. 808] and others are analyzed quantitatively.

At higher temperatures, dislocations may be torn from the impurity atoms which pin them. This gives rise to a hysteresis independent of frequency and of stress amplitude, but increasing rapidly with temperature. Analysis of the experimental observations leads to estimates of dislocation loop lengths and densities compatible with those derived from low-temperature internal friction and from other data such as etch pits and x-ray analysis.

At still higher temperatures, grain boundary slip as observed by Kê predominates, and is attributed to climbing of dislocations out of their glide planes by self-diffusion.

The effect of heavy cold work on damping is also discussed.

Reviewer believes this theory is an important contribution, but would welcome further evidence that pinning points are, in fact, impurity atoms and not other dislocations. Thermoelastic damping by moving dislocations. Eshelby [*Proc. roy. Soc. Lond.* (*A*) **197,** 396–416, 1949] might also have been considered.

F. R. N. Nabarro, South Africa

1639. Tielsch, H., and Boczek, A., A contribution to ultrasonic image conversion by thermal methods (in German), *Z. Phys.* **7,** 5, 213–218, May 1955.

A method is presented of obtaining a visible ultrasonic image by converting the wave energy into heat in a Buna or rubber film and using reversible thermocolors, such as salts of mercuric iodides. Temperature rise in the film is investigated theoretically and experimentally, and several photographs of the images are presented. Author suggests the technical application of the method, say, for the examination of metal sheets in roll work.

H. Tamaki, Japan

1640. Shinianskiĭ, L. A., Absorption of ultrasonic waves in rubber (in Russian), *Zh. tekh. Fiz.* **24,** 5, 851–853, May 1954.

1641. Torikai, Y., and Negishi, K., A simple method for the visualization of ultrasonic fields, *J. phys. Soc. Japan* **10,** 12, p. 1110 (Short Notes), Dec. 1955.

1642. Richardson, E. G., Industrial applications of ultrasonics, *Brit. J. appl. Phys.* **6,** 12, 413–415, Dec. 1955.

1643. Kapustin, A. P., Degassing of fluids in an ultrasonic field (in Russian), *Zh. tekh. Fiz.* **24,** 6, 1008–1011, June 1954.

1644. Fleischmann, W. L., and Rocha, H. A. F., Ultrasonic testing of small-diameter tubing with automatic recording equipment, *Trans. ASME* **78,** 1, 211–216, Jan. 1956.

Paper presents the progress made in the ultrasonic test equipment which now appears to make automatic testing of small-diameter tubing a distinct possibility. It then presents some cross sections, showing the type and size of discontinuities detected with this equipment in Type 347 stainless-steel tubing. Finally, the summary points out the inherent difficulties in judging the severity of the defect by the amplitude indications alone and the importance of the experience factor which is required to interpret the test results. From authors' summary

1645. Hiedemann, E. A., Metallurgical effects of ultrasonic waves, *J. acoust. Soc. Amer.* **26,** 5, 831–842, Sept. 1954.

This survey on the metallurgical effects of ultrasonic waves, including scientific and technological literature, deals with the following: (A) The techniques for the treatment of melts by mechanical vibrations. 1. Mechanical vibrators. 2. Electromechanical transducers. 3. Ponderomotive effects of induced electrical currents. (B) Metallurgical effects observed after ultrasonic treatment. 1. Reduction in grain size. 2. Degassing of melts. 3. Dispersion of other substances in melts. 4. Acceleration of rate processes. 5. Other metallurgical and related effects. From author's summary

1646. Fowell, L. R., and Korbacher, G. K., A review of aerodynamic noise, *Univ. Toronto Inst. Aerophys. UTIA Review* no. 8, 85 pp. + 48 figs., July 1955.

A detailed review is presented of the theoretical and experimental advances that have been made in the study of aerodynamic noise and in devising means for its suppression. The necessary background of laws and ideas from the field of acoustics is included. The theories for noise caused by subsonic disturbances, which may include turbulence fields in overchoked jets (Lighthill, Proudman), and for those noise sources peculiar to overchoked jets (Lighthill, Powell, Ribner), are considered. Experimental results are quoted complete with numerous graphs, notes on correlation of data for model and engine jets, and a comparison with theory. The results of attempts at noise suppression are discussed, noting both untried and extensively tested suggestions. A list of references is included. From authors' summary

1647. Vasilev, E. I., and Lopukhin, V. M., Theory of noise in traveling wave tubes (in Russian), *Zh. tekh. Fiz.* **24**, 5, 895–898, May 1954.

1648. Davis, D. D., Jr., Stokes, G. M., Moore, D., and Stevens, G. L., Jr., Theoretical and experimental investigation of mufflers with comments on engine-exhaust muffler design, *NACA Rep.* 1192, 47 pp., 1954.

See AMR **6**, Rev. 3180.

1649. Mokhtar, M., and Mahrous, M. A., The attenuation of sound in a turbulent atmosphere over a desert terrain, *Acustica* **5**, 3, 179–181, 1955.

The effect of atmospheric turbulence on the over-all sound intensity received downwind of a source at different distances from it and at various heights above the ground was investigated. The results revealed the occurrence of a zone of maximum intensity followed by another of minimum intensity at a higher level. The effect of "eddies" formed in the turbulent atmosphere on the diffraction pattern of the source was studied.

From authors' summary by M. J. P. Musgrave, England

Ballistics, Detonics (Explosions)

(See also Revs. 1359, 1527, 1633)

1650. Sakurai, A., Decrement of blast wave, *J. phys. Soc. Japan* **10**, 11, p. 1018, Nov. 1955.

Results of recent numerical study of blast-wave decay [Goldstine and von Neumann, AMR **9**, Rev. 798] are compared with other theoretical studies and with experimental data. Results are reduced to universal form of curve of $-d(\ln U)/d(\ln R)$ versus C/U, where U is speed of shock relative to undisturbed gas, C is speed of sound in undisturbed gas, and R is distance from origin of spherically symmetric wave front.

Theories of Sakurai and of Goldstine and von Neumann predict increased rate of shock-strength decay as shock strength increases. They agree with experimental results, in both trend and order of magnitude, for shocks with $C/U > 0.3$. For stronger shocks ($C/U < 0.3$), the theories do not reproduce experimental observation that rate of shock-strength decay decreases with increase of shock strength. A. H. Shapiro, USA

1651. Baillard, M. G., Exterior ballistics. Comparison between two classical methods of measuring precision of fire of small arms on vertical targets (in French), *Mém. Artill. fr.* **29**, 111, 159–179, 1955.

Author contrasts the "mean radius" criterion used in Anglo-Saxon nations with the quicker but superficially less attractive "height-plus-width" criterion used in France. The mean radius is the arithmetic mean of distances measured from the center of impact to the various points of impact. The height and width are those of the minimal vertical rectangle on the target, containing all points of impact. Author examines this new problem thoroughly, in light of theory and observed data, concluding that the French system is preferable. A. A. Bennett, USA

1652. Slawsky, Z. I., Michels, A., and Jacobs, S. J., The rapid expansion of compressed gases behind a piston, the effect of molecular interaction; Part I, Theory of the experiment; Part II, An apparatus for studying transient expansions, *Physica* **20**, 4, 210–222, 223–236, Apr. 1954.

In the first part of this paper, the isentropic simple wave expansion of a gas behind a suddenly released piston, initially at rest, is analyzed. For an initial pressure and temperature of 2500 psi and 495 R, respectively, and a given piston mass, the distance-versus-time curve is calculated for three different equations of state: (1) $pv = RT$; (2) $p(v - b) = RT$; and (3) the exact data for N_2. While the curves for (1) and (2) are identical, the piston travels, in case 3, about 15% less far than in cases (1) and (2) in the same time. Authors conclude that such an experiment may be used to calculate thermodynamic data: for instance, the effective molecular weight of a gas mixture.

In the second part, the apparatus for performing the above experiment is described in detail. The piston displacement versus time is measured with an accuracy of $2 \cdot 10^{-3}$ cm and 10^{-6} sec. The barrel of 50-cm length can contain a pressure of 54,000 psi.

Reviewer sees that this apparatus could be useful for the study of lag of thermodynamic equilibrium.

H. P. Eichenberger, USA

1653. Friedrich, H. R., and Dore, F. J., The dynamic motion of a missile descending through the atmosphere, *J. aero. Sci.* **22**, 9, 628–632, 638, Sept. 1955.

A method is presented for computing rapidly, yet accurately, the dynamic motion of a ballistic-type missile descending through the atmosphere. The equations of motion are separated into a set of "static" trajectory equations (zero angle of attack) and a set of "rotational" equations describing the oscillatory motion of the missile about its center of gravity. A transformation allows the rotational equations to be written in a manner analogous to the equation for an undamped oscillatng spring mass system with the mass equal to unity and a time variable spring constant. For given initial conditions this equation can be solved to obtain the envelope of maximum angle of attack. An additional transformation allows the calculation of the complete oscillatory motion at any time during the trajectory as a function of the maximum angle of attack at that time.

This solution shows that the maximum angle of attack of a missile descending through the atmosphere at relatively constant speed is reduced even when the aerodynamic damping is neglected.

From authors' summary by E. Leimanis, Canada

Soil Mechanics, Seepage

(See also Revs. 1367, 1376, 1377, 1411, 1660)

1654. Nishida, Y., Computation of earth pressure exerted on a vertical wall (in German), *Bauingenieur* **30**, 9, 320–322, Sept. 1955.

As an approximation, the earth pressure upon a vertical retaining wall is calculated with the theory of elasticity, assuming that the problem is of axial symmetry in which the axis of z is in a wall surface.

The distributions of earth pressure formerly measured and proposed by other authors are compared with the result, which seems to give an agreement. But the reviewer thinks that this approximation does not give an explanation beyond a qualitative one.

T. Mogami, Japan

1655. **Shield, R. T., On Coulomb's law of failure in soils,** *J. Mech. Phys. Solids* **4,** 1, 10–16, Oct. 1955.

This work is an extension of work done by author and by Drucker and Prager on the Coulomb law of failure in soils. Tresca criterion was used. Assuming a perfect plastic material, author discusses the associated flow rule. The dissipation function is obtained and applied to limit analysis theorems. As an example, upper and lower bounds are computed for a rectangular punch.

As the author states, the results are correct for the ideal plastic materials but not necessarily applicable to real soils.

W. Sylwestrowicz, USA

1656. **Capper, P. L., Structural aspects of road design,** *Struct. Engr.* **34,** 2, 56–61, Feb. 1956.

Paper gives a short account of the application of soil mechanics to road design. The principles by which wheel loads are transmitted to and resisted by the subgrade are discussed. Several methods of pavement design are reviewed, in which the necessary pavement thickness is related to the soil properties. Some experimental results are given to show the correlation of certain soil properties and their variation with moisture content. Finally, a brief reference is made to cement stabilization.

From author's summary

1657. **Artobolevskii, I. I., Bessonov, A. P., and Raevskii, N. P., Dynamic line diagrams of soil pressure on a pile driven by the vibratory method** (in Russian), *Izv. Akad. Nauk SSSR Otd. Tekh. Nauk* no. 7, 116–121, July 1954.

Micromeritics

(See also Revs. 1541, 1580)

1658. **Rouse, H., and Dodu, J., Turbulent diffusion across a density discontinuity** (in French and English), *Houille blanche* **10,** 4, 522–532, Aug.-Sept. 1955.

An experimental investigation was made of the effect on two superposed liquids of slightly different density of placing a vertically oscillating agitator in the upper liquid; otherwise the two liquids were at rest. Dimensional analysis is applied to the experimental results and relations are obtained among a Froude number, a Reynolds number, a diffusion parameter, and a time parameter. The two main results are: "(a) turbulence generated in one of the two fluid strata which are otherwise at rest neither penetrates the second stratum nor produces a buffer layer between the two; rather, fluid is entrained from the surface of the nonturbulent stratum and diffused throughout the turbulent zone"; and (b) "so long as the turbulence is produced by some external mechanism, for small density differences a limit of stability comparable to that which characterizes interfacial shear is not to be expected; instead, there appears to be a finite rate of diffusion across the interface for any Froude and Reynolds numbers."

N. Tetervin, USA

1659. **Schneebeli, G., Experiments on the range of validity of Darcy's law and the appearance of turbulence in a filtering flow** (in French), *Houille blanche* **10,** 2, 141–149, Mar./Apr. 1955.

1660. **Barenblatt, G. I., On limiting (automodeled) motions in the theory of nonstationary filtration of gas in a porous medium and in the theory of boundary layer** (in Russian), *Prikl. Mat. Mekh.* **18,** 4, 409–414, July/Aug. 1954.

1661. **Hahnemann, H. W., Generalized representation of pressure loss in wire sifter** (in German), *ZVDI* **97,** 30, 1061–1062, Oct. 1955.

Geophysics, Meteorology, Oceanography

(See also Revs. 1529, 1540, 1551, 1553, 1608, 1643, 1649, 1658)

1662. **Burgers, J. M., Rotational motion of a sphere subject to visco-elastic deformation, Parts I–III,** *Proc. k. Ned. Akad. Wet. (B)* **58,** 4, 219–237, 1955.

Approximate solution is given of dynamical problem introduced by T. Gold [*Nature* **175,** 4456, 526–529, Mar 1955] of influence of material of the earth on polar wandering. Model taken is a body which, in absence of stress, is spherical, subject to centrifugal force, and composed of matter possessing permanent stiffness in parallel with a stiffness which relaxes exponentially. It is first assumed, and then verified, that the change of form during polar wandering does not appreciably affect the components of angular momentum of the body. The case of small oscillations of the axis of rotation about a fixed direction, corresponding to very slow relaxation of stress, is first treated, followed by that of intermediate relaxation, which leads to a damped nutation. The more complicated case of polar wandering caused by small impressed changes in the moments of inertia is then treated, and Gold's assumption on the speed of polar wandering is verified. The final case considered is that of a change in the moments of inertia taking place in a time large compared with the damping time of nutation, but small compared with the time scale of polar wandering.

F. R. N. Nabarro, South Africa

1663. **Nakagawa, Y., and Frenzen, P., A theoretical and experimental study of cellular convection in rotating fluids,** *Tellus* **7,** 1, 1–21, Feb. 1955.

The stabilizing and cell-shape changing influence of a basic rotation upon thermally produced cellular circulations are developed in a first-order perturbation theory. The theory obtained for ordinary convection is compared with laboratory experiments with rotating cylinders of water, and the existence of an overstable oscillating mode is demonstrated experimentally. Numerical results and consideration of models indicate possible application of the theoretical results to atmospheric convention.

From authors' summary by D. M. Vestal, Jr., USA

Lubrication; Bearings; Wear

(See also Revs. 1454, 1487, 1630)

1664. **Feng, I-M., and Chang, C. M., Critical thickness of surface film in boundary lubrication,** ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-84, 3 pp.

Authors present results of experiments designed to show existence of critical thickness of surface film above which metallic wear drops to zero. Thickness of adsorbed film on surfaces of wear specimens was controlled by regulating vapor pressure of ethyl alcohol in air surrounding specimens. For heavy loads, a plot of wear versus ethyl alcohol vapor pressure was similar in shape to a Type II adsorption isotherm, as predicted by the authors. At light load, the curve broke sharply at some critical value of vapor pressure, the wear dropping to zero. These results are explained in terms of a "plastic roughening" theory of

friction postulated earlier by one of the authors. It is thought that surface damage is largely transfer of metal from one surface to another with thin surface films, more loose wear particles being formed as the films increase in thickness until a critical thickness is reached when the metallic asperities are fully separated by the contaminating layer and no wear occurs.

C. D. Strang, Jr., USA

1665. Hagg, A. C., and Sankey, G. O., Some dynamic properties of oil-film journal bearings with reference to the unbalance vibration of rotors, ASME Ann. Meet., Chicago, Ill., Nov. 1955. Pap. 55—A-45, 5 pp.

Elastic and damping forces developed in oil films of a 150° partial bearing and a pivoted pad journal bearing are determined experimentally. An example of the influence of these properties of oil films on magnitude of unbalance vibration and critical speeds of rotors is given. W. J. Anderson, USA

1666. Rabinowicz, E., The relation between friction and wear for boundary-lubricated surfaces, *Proc. phys. Soc. Lond* (*B*) **68,** part 9, no. 429 B, 603–608, Sept. 1955.

Author assumes that all the friction between boundary-lubricated surfaces is due to adhesion between the metals or to shear resistance of the lubricant, while all the wear at the metallic junctions is in the form of welded metal fragments. This assumption leads to an equation stating that the $^2/_3$ power of metal transfer (in g per cm track) is proportional to the difference between the friction coefficient f and some constant f_1, depending for the major part on the chemical structure of the lubricant and only slightly on the temperature. To check this law, some metal transfer experiments and some loose experiments were carried out.

Metal transfer experiments were made according to an earlier described technique, with a hemispherically ended radioactive copper rider sliding on a flat copper surface (load: 2000 g, speed: 0.01 cm/sec) and different long-chain compounds having about the same value of f_1. In plotting the log of the difference $(f - f_1)$ in function of the log of metal transfer for 250 points, a good agreement was found with a straight line having a slope of $^2/_3$. Loose wear experiments were carried out on a $^1/_4$-in. hemispherically ended rider of titanium producing a circular track of 5-cm diam on a titanium flat. Different lubricants were used. The titanium flat and rider were weighed before and after each run and the loss in weight was calculated. f was measured and f_1 was estimated from previous work. A plot of $^2/_3$ power of the wear rate in function of the friction coefficient gives a very satisfactory fit. Data for experiments made earlier by Neely were recalculated and show results consistent with the postulated theory.

In a discussion, attention is laid on some restrictions of the proposed law. D. De Meulemeester, Belgium

1667. Johnson, K. L., Surface interaction between elastically loaded bodies under tangential forces, *Proc. roy. Soc. Lond.* (*A*) **230,** 1183, 531–548, July 1955.

Paper describes measurements of micro-displacements of hard steel balls in contact with a hard steel flat surface due to tangential forces less than that of limiting friction. Tangential forces are both steady and oscillatory. Normal loads and ball diameters are varied. In the oscillatory case, the energy loss is also determined by means of the method of decaying oscillations.

The results for displacements and energy loss are compared with the theoretical work of Mindlin et al. [AMR **3,** Rev. 2592; **6,** Rev. 3659]. The measured displacements are in close agreement with theory, while for the energy loss the measured values have the same order of magnitude as the theoretical ones.

H. Bergh, Holland

1668. Moore, C. C., and Jones, F. C., Opening characteristics of high-speed bearings at high oil flow rates, Second Ann ASLE-ASME Conf., Indianapolis, Ind., Oct. 1955. Pap. 55—LUB-10, 6 pp. + 5 tables, 8 figs.

Using 1005 petroleum oil as a lubricant, 202-, 203-, and 204-size ball bearings were operated in a speed range of between 0.6 to 1.7 $\times 10^6$ DN value. Under lightly loaded conditions, oil flow rates were varied between 1 and 500 range while friction power, oil-inlet, and outer-race temperature were measured at speeds between 30,000 and 100,000 rpm. From authors' summary

1669. Dubois, G. B., and Ocvirk, F. W., Analysis of plain journal bearings, *Prod. Engng.* (*Product Design Handbook for* **1956**) **26,** 11, F18–F22, Oct. 1955.

1670. Cordiano, H. V., Cochran, E. P., and Wolfe, R. J., Effect of combustion resistant hydraulic fluids on ball bearing fatigue life, Second Annual ASLE-ASME Conf., Indianapolis, Oct. 1955. Pap. 55—LUB-21, 6 pp. + 5 figs.

Marine Engineering Problems

1671. Dehnert, H., Locks and elevators [Schleusen und Hebewerke], Berlin/Göttingen/Heidelberg, Springer-Verlag, 1954, viii + 340 pp. DM 45.

This handbook on navigation locks and elevators for ships is one of a series of books for civil engineers, well known in Germany. Edited with usual care and precision it presents a large summary of knowledge in this particular field, and also an interesting historical review. Its contents are: Purpose, types, and history of locks; filling and emptying of lock chambers; planning of locks, size, and construction process; sluices and gates: types and operation; valves; design of a lock: chamber, heads, channel, inlets, guides; machinery and its operation; efficiency; water conservation: reserve and storage basins; elevators: types—vertical and sloping; mechanical details. Extensive bibliography contains 457 titles. A very useful book for hydraulic and transportation engineers. S. Kolupaila, USA

1672. Gover, S. C., and Olson, C. R., A method for predicting the torque of semibalanced centerline rudders on multiple-screw ships, *David W. Taylor Mod. Basin Rep.* no. 915, 15 pp., Nov. 1954.

An empirical method is presented for predicting the torque required for semibalanced rudders not directly in the propeller slipstream. An example of the method is given and theory, full-scale trials, and model test results are compared.

From authors' summary

1673. Schuster, S., New instruments for ship model tests, *European Shipbldg.* **6,** 3, 145–149, 1954.

1674. Steneroth, E., Some remarks on calculation of the transverse strength of tankers, *European Shipbldg.* **5,** 3, 136–142, 1954.

1675. Benford, H., The control of yaw in towed barges, *Intern. Shipbldg. Progr.* **2,** 11, 296–308, 1955.

The aim of this paper is to tell what has been learned by experiment about yaw control. From author's summary